

Clear and liquid: The interaction of firm disclosure and trader competition*

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September 16, 2018

Abstract

In an economy where traders absorb information partially, we study the effect of disclosure accuracy and disclosure clarity on financial markets. Accuracy measures how precisely a disclosure identifies the firm's fundamentals, whereas clarity measures how well traders understand the disclosure. Increasing clarity promotes competition among informed traders more so than it increases the information asymmetry between informed traders and market makers. Increasing accuracy has the opposite effect. Trading volume increases monotonically in clarity, while liquidity has a U-shaped relationship with clarity if competition among traders is intense. Volume and liquidity both decrease in accuracy due to adverse selection among different market participants. Moreover, traders' attention to disclosure is hump-shaped in clarity, but increasing in accuracy. Trading profits mimic the patterns of attention, but only under intense competition. Overall, our results suggest that the current trend of increasing complexity in firm disclosures is detrimental to liquidity and volume.

JEL: D53, G14, M41, D82.

Keywords: competition, bounded rationality, accuracy, clarity.

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1 Introduction

“For more than forty years, I’ve studied the documents that public companies file. Too often, I’ve been unable to decipher just what is being said [...] stilted jargon and complex constructions are usually the villains.”

—Warren Buffett, in “A Plain English Handbook” (SEC, 1998)

Traditional models of firm disclosure assume that participants of financial markets can fully absorb all the information that firms disclose.¹ In reality, incorporating complex accounting information into investors’ trading decisions requires the use of scarce cognitive resources. As Dyer et al. (2017) estimate, understanding the median 10-K may require as much as 21 years of formal education. Ongoing regulatory initiatives—such as the “Plain English Rule” of the Securities and Exchange Commission (SEC) and the “Cutting Clutter” rule of the Financial Reporting Council—further highlight how difficult it is for real-world traders to fully comprehend disclosed information.

We consider a model that incorporates limits to information absorption by assuming that traders observe firm disclosures with noise. Disclosing information to financial markets thus involves two kinds of noise: one arising from what firms release about their fundamentals, and another, arising from traders’ imperfect comprehension of what firms release. These two kinds of noise correspond to two dimensions of firm disclosure. The first dimension, which we call accuracy, captures what fully rational models designate as disclosure precision. For example, a Form 10-K is more accurate if it identifies the firm’s fundamentals more precisely. The second dimension, which we call clarity, captures how easy it is for traders to understand

¹See, for example, Diamond and Verrecchia (1991) and Easley and O’Hara (2004). Exceptions include Holthausen and Verrecchia (1990) and Indjejikian (1991).

the disclosed information.² For example, a Form 10-K is clearer if, in the words of Warren Buffet, it is easier “to decipher.”

Embedding these two dimensions of disclosure into a standard trading model we ask the following question. How do markets react when firms increase disclosure accuracy and disclosure clarity? As we show in this paper, market liquidity, trading volume, and traders’ attention to disclosure are affected in qualitatively different ways by accuracy and clarity.

Our market structure is based on Kyle (1985). The economy consists of liquidity traders, informed traders, and a firm which provides a noisy signal of its fundamentals to the informed traders. We call this signal a firm disclosure. The amount of noise in the firm disclosure determines the disclosure accuracy—lower noise variance corresponds to higher accuracy.

Building on recent literature on costly information processing (Myatt and Wallace, 2012; Pavan, 2016), we assume that each informed trader observes the firm disclosure with idiosyncratic processing noise which he can reduce by paying attention. The extent to which a trader’s attention decreases his noise depends on disclosure clarity—clearer disclosures are easier to understand, and result in lower noise for each informed trader. For simplicity, we assume that the market maker does not observe the firm disclosure, as in Kim and Verrecchia (1994).³

Our first result is that liquidity decreases in accuracy, but it has a U-shaped relationship with clarity if competition among traders is sufficiently high. Higher accuracy provides more information to the informed traders, and it thus increases the information asymmetry between them and the market maker. Higher information asymmetry in turn increases the adverse-selection problem between the traders and the market maker, and, as is standard in

²Our terminology of accuracy and clarity follows Myatt and Wallace (2012), who study attention allocation in a beauty contest with multiple information sources.

³This assumption is consistent with that sophisticated traders (e.g. hedge funds) are better in processing information than market makers.

many trading models, it reduces liquidity (e.g., O’Hara, 1995).

The effect of clarity on liquidity depends on the tradeoff between two opposite forces. On the one hand, higher clarity decreases the processing noises of the traders, which increases their information advantage over the market maker. Following existing literature we call this force the “adverse-selection effect.” On the other hand, higher clarity also increases the correlation among the traders’ signals, which diminishes the traders’ information advantage over the market maker and intensifies the competition among the traders. We call this the “competition effect.” At low levels of clarity the adverse-selection effect dominates. The information asymmetry between the traders and the market maker thus increases with higher clarity, and liquidity consequently decreases. Nevertheless, the competition effect dominates when there are sufficiently many traders and clarity is sufficiently high. In that case, the traders’ information advantage decreases on balance with higher clarity, and liquidity increases.

There is thus a certain value of disclosure clarity that firms must overcome to improve liquidity. We call this value the “clarity threshold”—increasing clarity below this threshold has a negative effect on liquidity. We show that the clarity threshold decreases in the number of traders but increases in accuracy. Since competition is the main driver behind the positive effect of clarity on liquidity, a larger number of traders decreases the clarity threshold because it increases competition. In contrast, higher accuracy reduces the variation of the common noise in the traders’ signals, which makes these signals driven more by idiosyncratic noises. As such, higher accuracy reduces the correlation among the traders’ signals, which unravels the competition effect and in turn increases the clarity threshold. Higher accuracy thus impacts liquidity in two different ways: by its direct negative effect on liquidity, and by its indirect effect of increasing the clarity threshold.

Our second result is that trading volume decreases in accuracy but increases in clarity. This result thus reaffirms existing literature which shows that increasing accuracy has a detrimental effect on volume (Kim and Verrecchia, 1994). Nevertheless, in our model volume increases in clarity because, as we discuss above, clearer information makes the traders compete more intensely. In intuitive terms, the traders gain a better private understanding of what the firm discloses when clarity increases. The traders then trade more aggressively, thereby increasing volume.

Our third result is about how trader attention endogenously responds to firm disclosure. We show that trader attention increases in accuracy, but it is hump-shaped in clarity. The former relation is a straightforward consequence of that higher accuracy reduces uncertainty about the firm's fundamentals, which increases the traders' incentives to pay more attention to the firm disclosure. The intuition behind the latter relation is more subtle.

When clarity is very low, the net benefit that the traders derive from paying attention to the disclosure is low relative to their cost. When clarity is very high, the disclosure is so easy to understand that the traders do not need to pay much attention to it. As such, trader attention is larger at intermediate values of clarity, but it is smaller at high and low values of clarity.⁴ Myatt and Wallace (2012) and Pavan (2016) report similar results in the context of beauty-contest games. Our results show that the relation between endogenous attention and clarity also holds in financial markets without coordination motives in traders' preferences.

Our framework offers a potential reconciliation between existing theoretical predictions and empirical regularities about the impact of firm disclosure on liquidity. Prior research suggests that releasing new information to the market may worsen the information asymmetry between sophisticated and unsophisticated market participants, and that it may reduce

⁴This result also distinguishes our model from classic information acquisition (Verrecchia, 1982; Indjejikian, 1991). See the Internet Appendix for details.

liquidity. And yet, the vast majority of empirical studies show that firm disclosure reduces the bid-ask spread and improves liquidity (Lang et al., 2012; Balakrishnan et al., 2014).

Our model suggests that if releasing new information to the market increases the correlation of dividend expectations among competing sophisticated participants—as it would by improving disclosure clarity—then the information advantage that sophisticated participants hold over unsophisticated ones weakens, and liquidity improves. In addition, our predicted relationship between clarity and liquidity is consistent with evidence in Lang and Stice-Lawrence (2015) that bid-ask spreads are positively associated with the Gunning Fog index.

1.1 Literature review

Our paper relates to the wider theme of attention as a scarce cognitive resource (see Simon (1971) and Kahneman (1973) for seminal work). Within the accounting literature, there is growing evidence that individual traders often fail to incorporate value-relevant information into trading decisions (Hirshleifer et al., 2008; Ayers et al., 2011). Several studies indicate that more complex disclosures are costlier to process, and that they are associated with lower trading volume, lower ownership among small investors (Miller, 2010; Lawrence, 2013), lower price efficiency (You and Zhang, 2009; Callen et al., 2013), and higher idiosyncratic volatility (Loughran and McDonald, 2014).

The theory literature on how firm disclosure affects financial markets documents two countervailing forces (Diamond and Verrecchia, 1991; Kim and Verrecchia, 1994). First, disclosure may reduce information asymmetry by revealing to unsophisticated traders some information which is new to them, but is already known by sophisticated traders. Second, disclosure may increase information asymmetry even if it provides information that is new

to all traders, because of the advantage that sophisticated traders may have in processing new information.

Diamond and Verrecchia (1991) emphasize the former force. Kim and Verrecchia (1994) discuss both forces, showing that disclosure increases liquidity but it decreases trading volume. Our paper contributes to this literature in two ways. Instead of considering disclosure as a monolithic construct, we separate it into two dimensions—accuracy and clarity—and we examine how they each affect market behavior. We show that these two dimensions have economically different effects on markets. Moreover, we show that disclosing clearer information improves liquidity and volume despite that sophisticated traders have an advantage in processing new information.

In the context of coordination games, Myatt and Wallace (2012) and Pavan (2016) study how accuracy and clarity influence the attention that agents pay to multiple sources of information. They show that when beauty-contest motives are sufficiently strong, agents pay attention only to the clearest sources of information, even if these sources have poor accuracy. We have only one source of information, the firm, and we do not embed any coordination motives in our traders’ utility functions.

We use a model based on the one-period version of Kyle (1985) with competitive traders. There are three settings related to ours. Holden and Subrahmanyam (1992) is a dynamic model with many risk-neutral traders, who observe the fundamental without any noise. Subrahmanyam (1991) discusses a one-period setting with many risk-averse traders, all of whom observe the same noisy signal perfectly. He shows that with risk aversion liquidity can be U-shaped in accuracy—our traders are risk-neutral, and liquidity is decreasing in accuracy but U-shaped in clarity. In a dynamic economy with risk-averse traders and exogenous covariance structure, Foster and Viswanathan (1996) show that a “waiting game” emerges

with a small number of traders. In a static economy, we explore the effect of disclosure accuracy and clarity on trading; we show that profits and market impact with a large number of traders are different from those with a small number of traders.

Holthausen and Verrecchia (1990) and Indjejikian (1991) study rational expectations models à la Hellwig (1980), with price-taking traders who interpret the firm disclosure with idiosyncratic private noise. The private noise in Holthausen and Verrecchia (1990) is exogenous, and even though the noise in Indjejikian (1991) is endogenous, the firm disclosure has only one dimension, which corresponds to accuracy in our setting. Our traders are strategic, and we study the effect of both accuracy and clarity on liquidity and volume.

2 The model

Our economy has N informed risk-neutral traders, one competitive risk-neutral market maker, and a block of liquidity traders. There is one risky asset, which we call the firm, with liquidating value $v \sim \mathcal{N}(0, \tau_v^{-1})$.

There are three dates, $t = 1, 2, 3$. At $t = 1$ the firm makes a disclosure

$$f = v + \phi, \tag{1}$$

where $\phi \sim \mathcal{N}(0, \alpha^{-1})$, independently of v . By paying attention $z_i \in (0, \infty)$ to the firm disclosure f , informed trader i observes a private signal

$$s_i = f + \varepsilon_i = v + \phi + \varepsilon_i, \tag{2}$$

where $\varepsilon_i \sim \mathcal{N}(0, (\kappa z_i)^{-1})$, identically distributed over i , and independently of v and ϕ . The

attention cost of trader i is

$$C(z_i) = \frac{1}{2}cz_i^2, \quad (3)$$

with $c > 0$.

We call α the *accuracy*, and κ the *clarity* of the firm disclosure. Accuracy measures how precisely the disclosure f identifies the future liquidating value of the firm. Clarity measures how easy it is for each trader to understand what the firm means when it discloses f .⁵

At $t = 2$ the informed traders and the liquidity traders submit their orders to the market maker. Each informed trader $i = 1, \dots, N$ submits order x_i that maximizes his expected profit conditional on his information. The liquidity traders as a whole demand $\psi \sim \mathcal{N}(0, \tau_\psi^{-1})$.

The market maker does not observe the firm disclosure or the traders' signals, but he observes the total order flow. He sets the price p to be his expectation of the firm value conditional on order flow, as

$$p = E \left[v \mid \sum_{j=1}^N x_j + \psi \right]. \quad (4)$$

At $t = 3$ the liquidating value v is realized and all traders consume their profits.

We define expected trading volume V (volume, for short) as

$$V = \frac{1}{2}(V_m + V_c), \quad (5)$$

where V_m is the expected volume of trade between the market maker and all the other traders grouped together,

$$V_m = E \left| \sum_{j=1}^N x_j + \psi \right|, \quad (6)$$

⁵See also Myatt and Wallace (2012). As Pavan (2016) discusses, clarity—“transparency” in his context—is the rate of return of attention to the signal f .

and V_c is the expected volume of trade that is crossed between the informed and liquidity traders,

$$V_c = E \sum_{j=1}^N |x_j| + E |\psi|. \quad (7)$$

Our definition of volume follows Admati and Pfleiderer (1988), Foster and Viswanathan (1990), O’Hara (1995), Vives (2008), and others.

2.1 Asset-market equilibrium

We conjecture that the demand of each informed trader i is linear in his private signal,

$$x_i = \beta_i s_i, \quad (8)$$

and that the asset price is linear in aggregate demand,

$$p = \lambda \left(\sum_{j=1}^N x_j + \psi \right). \quad (9)$$

The parameter λ (“market impact”) measures illiquidity, and β_i (“trading intensity”) measures how aggressively trader i uses his information. We adopt λ^{-1} as our measure of liquidity.

Lemma 1 *For given attention levels z_1, \dots, z_N , there exists a unique equilibrium in which x_1, \dots, x_N are as in (8) and p is as in (9), with*

$$\beta_i = \frac{1}{\lambda \nu} \frac{h_i}{H}, \quad (10a)$$

and

$$\lambda = \frac{1}{\nu} \frac{1}{\sum_{j=1}^N \beta_j}, \quad (10b)$$

where

$$\nu = 1 + \frac{\tau_v}{\alpha} + \frac{1}{H}, \quad (10c)$$

$$\sum_{j=1}^N \beta_j = \left[\tau_\psi \left(\frac{1}{\tau_v H} - \frac{\sum_{j=1}^N \frac{h_j^2}{\kappa z_j}}{H^2} \right) \right]^{-\frac{1}{2}}, \quad (10d)$$

$$h_j = \frac{1}{1 + \frac{\tau_v}{\alpha} + 2 \frac{\tau_v}{\kappa z_j}}, \quad (10e)$$

$$H = \sum_{j=1}^N h_j. \quad (10f)$$

As (10b) shows, liquidity λ^{-1} is the product of two terms, ν and $\sum_{j=1}^N \beta_j$. The ν term captures disclosure noise effects. To see why, let us suppose there is only one informed trader, as in Kyle (1985). The expression in (10b) then becomes

$$\lambda^{-1} = \nu \beta, \quad (11)$$

where

$$\nu = 2 \left(1 + \frac{\tau_v}{\alpha} + \frac{\tau_v}{\kappa z} \right). \quad (12)$$

If the trader's signal is perfectly accurate and perfectly clear (that is, if $\alpha \rightarrow \infty$ and $\kappa \rightarrow \infty$) we recover the inverse relationship between market impact and trading intensity in Kyle (1985), where

$$\lambda^{-1} = 2\beta. \quad (13)$$

Comparing our expression for liquidity in (11) to that in (13) we can see that ν captures

noise effects from extending Kyle (1985) to our setting with firm disclosure. We thus refer to the first term in the expression for liquidity, shown explicitly in (10c), as the “noise term.”⁶

The second term in (10b),

$$\sum_{j=1}^N \beta_j, \tag{14}$$

is an aggregate version of the trading intensity in Kyle (1985), and it captures effects of trader competition. We refer to it as the “competition term.”

2.2 The effect of clarity and accuracy

Equation (10b) shows that increasing either accuracy or clarity has a direct effect on liquidity. Holding trading intensity fixed, disclosing more information to the traders increases their market impact because the total order flow contains more information. In particular, as the firm discloses more information, the price becomes less noisy and more responsive to information about the fundamental. Because information about the fundamental enters the price through the traders’ orders, it follows that the extent to which a trader can move the price increases.

An additional effect on market impact enters through aggregate trading intensity. If the firm discloses more information to the traders, the traders act more aggressively because their signals become less noisy. This is true no matter which dimension of disclosure improves—either more accurate information or clearer information reduces the noise in traders’ signals. As we discuss next, however, the overall effects of improving disclosure in terms of accuracy and clarity are very different. To simplify exposition we assume that trader attention is exogenous and symmetric, while we endogenize attention in the following section.

⁶(10c) is a further extension of (12) to account for many traders.

Theorem 2 *With exogenous symmetric attentions*

- (i) *the noise term decreases in clarity, whereas the competition term increases in clarity,*
- (ii) *for $N \leq 3$, liquidity decreases in clarity, but for $N > 3$, liquidity is U-shaped in clarity,*
and
- (iii) *volume increases in clarity for $N > 1$.*

Our first two results in Theorem 2 confirm the intuition we portray above. Nevertheless, these two results highlight two different channels for how clarity affects liquidity. On the one hand, increasing clarity decreases the noise term, and it decreases liquidity. On the other hand, increasing clarity also increases the competition term, which reduces market impact and improves liquidity. Whether liquidity improves or deteriorates therefore depends on the relative strength of the noise and competition channels.

In particular, as we can see in our second result, what determines which effect dominates is the number of traders. To show how the number of traders affects liquidity, we consider its effect on the competition term and on the noise term separately.⁷ The competition term is

$$\sum_{j=1}^N \beta_j = \left(\frac{N}{\tau_\psi \left(\frac{1}{\tau_v} + \frac{1}{\alpha} + \frac{1}{\kappa z} \right)} \right)^{1/2}. \quad (15)$$

We can see that it increases in clarity, and that this effect becomes more pronounced with more traders. The noise term is

$$\nu = \left(1 + \frac{1}{N} \right) \left(1 + \frac{\tau_v}{\alpha} \right) + \frac{2}{N} \frac{\tau_v}{\kappa z}. \quad (16)$$

⁷See Lemma A.1 of the Appendix for details on the components of liquidity.

In this expression clarity appears with an inverse order in the number of traders. Increasing clarity thus decreases the noise term, but not as strongly as it increases the competition term. In other words, the effect of clarity in the noise term is relatively weaker than that in the competition term. This is intuitive, because the price aggregates the traders' signals, and thus, due to a law-of-large-numbers effect, the aggregate signal does not respond to the idiosyncratic processing noises as strongly as to the systematic firm noise.⁸

In sum, the competition term increases in clarity and more so with more traders, whereas the noise term decreases in clarity and less so with more traders. Taken together, these two effects make liquidity a U-shaped function of clarity.

In terms of the information asymmetry between the market maker and the informed traders, we would expect the adverse-selection problem to worsen for the market maker when he faces more informed traders. Nevertheless, with more traders there is more competition, and the adverse selection between the traders and the market maker is less severe. Moreover, because higher clarity makes the traders' signals more correlated, when clarity increases the informed traders observe less differentiated information.⁹ Higher clarity therefore weakens the informed traders' advantage over the market maker, it intensifies competition, and it mitigates adverse selection.

Trading volume exhibits similar effects to those in liquidity, but with a different overall

⁸Foster and Viswanathan (1996) show that for small numbers of traders the degree of monopoly power associated with differentiated information makes traders more conservative, in an intertemporal manner. In our static context, we show that for large numbers of traders the monopoly of differentiated information is diminished.

⁹In a symmetric equilibrium, the pairwise correlation of the signals of any two traders i and j conditional on the fundamental v is $\text{Corr}(s_i, s_j | v) = \kappa z / (\kappa z + \alpha)$, which increases in clarity.

outcome. As we show in Lemma A.2 of the Appendix, the volume is

$$V = \frac{1}{\sqrt{2\pi\tau_\psi}} \left[\sqrt{\frac{\tau_\psi}{\tau_v} \lambda^{-1} \sum_{j=1}^N \beta_j + \sqrt{N} + 1} \right] = \frac{1}{\sqrt{2\pi\tau_\psi}} \left[\sqrt{\frac{\tau_\psi}{\tau_v} \nu \sum_{j=1}^N \beta_j + \sqrt{N} + 1} \right]. \quad (17)$$

We can see that volume is directly related to liquidity. Holding the competition term fixed, we thus expect volume to have a U-shaped relationship with clarity. Nevertheless, the total effect of higher clarity on volume is positive because there is a stronger effect from the competition term.¹⁰ This additional effect overwhelms that liquidity decreases for small values of clarity, and it makes volume increase in clarity overall.

That liquidity is U-shaped in clarity implies that there is a point where liquidity reaches a minimum. The value of clarity κ_* that corresponds to this minimum liquidity is the clarity threshold. Any effort by the firm to improve its liquidity by increasing its disclosure clarity is counterproductive unless its existing clarity exceeds κ_* .

Corollary 3 *The clarity threshold κ_* decreases in the number of traders and increases in accuracy.*

As we explain above, the competitive aspect of trading is the main reason why clarity has a positive effect on liquidity. In this sense, clarity and competition act as substitutes of each other, and thus a disclosing firm in a market with more traders faces a lower clarity threshold.

In contrast, the clarity threshold increases in accuracy. This happens because higher accuracy reduces the common noise in the traders' signals, which makes them driven relatively more by their idiosyncratic components, which, in turn, subdues competition among

¹⁰After substituting the expression for λ^{-1} from (10b) into (17), the noise term appears in (17) under the square root, whereas the competition term appears outside the square root.

the traders. To countervail this side effect, the firm must make its disclosure clearer.

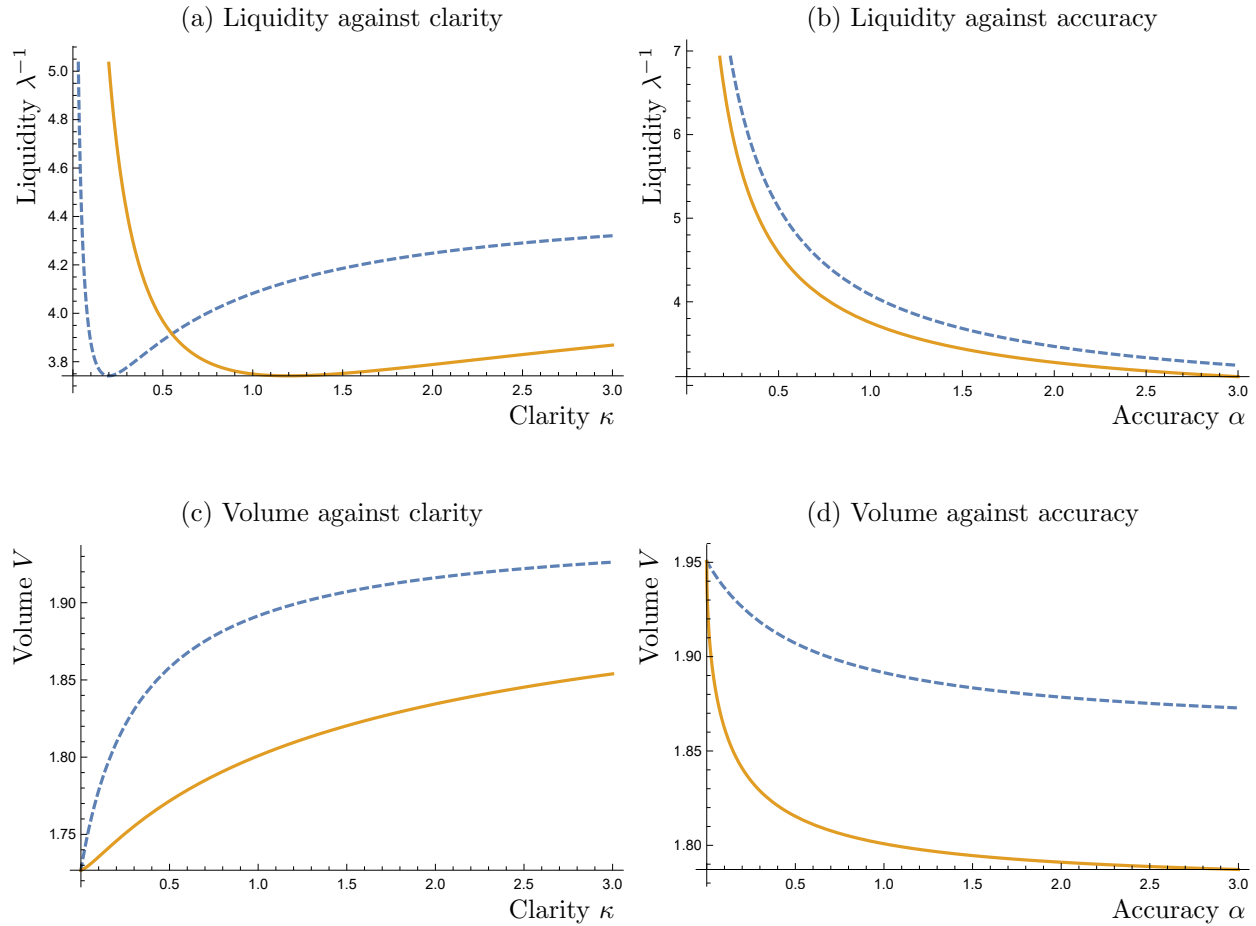


Figure 1: Liquidity λ^{-1} and expected trading volume V as functions of the characteristics of firm disclosure, for exogenous attention (in dashed blue) and for endogenous attention (in solid orange). On the left we show liquidity and volume against clarity κ holding $\alpha = 1$. On the right we show liquidity and volume against accuracy α holding $\kappa = 1$. Our remaining parameters are $N = 8$, $\tau_\psi = 1$, $\tau_v = 1$, $c = 1$, where for the graphs with exogenous attention we hold $z = 1$.

The effects of accuracy on the competition term and the noise term are analogous to those for clarity. The relative strengths of these effects, however, are different enough to produce a different set of comparative statics.

Theorem 4 *With exogenous symmetric attentions*

(i) *the noise term decreases in accuracy, whereas the competition term increases in accuracy,*

(ii) *liquidity decreases in accuracy, and*

(iii) *volume decreases in accuracy for $N > 1$.*

What differentiates the results of Theorems 2 and 4 is that unlike for clarity, there is no law-of-large-numbers effect for accuracy, because accuracy is the precision of the common noise in traders' signals. Consequently, the effect of accuracy on the noise term is strong even for large numbers of traders.¹¹ This fact makes the effect of accuracy through the noise term strong enough to overwhelm the effect of accuracy through the competition term, both in liquidity and in volume. Figure 1 shows an illustration. Our next result discusses trader profits.

Lemma 5 *With exogenous symmetric attentions, trader profit increases in accuracy. It also increases in clarity for $N \leq 3$, but it is hump-shaped in clarity for $N > 3$.*

Improving accuracy increases trader profits because it reduces the uncertainty about the fundamental of the firm. For similar reasons, improving clarity increases trader profits, but only with a small number of traders. As the number of traders increases, improving clarity has two effects. Not only does it reduce the uncertainty about the firm fundamental, which increases profits, but it also intensifies competition, which reduces trader profits. As a result profits are hump-shaped in clarity.

¹¹In (16) the scale of competition for accuracy is $1 + 1/N$, but for clarity it is $1/N$.

2.3 Attention equilibrium

At $t = 0$ each trader selects his attention z_i to maximize his ex-ante expected profit net of attention costs,

$$\max_{z_i} E [E [x_i (v - p) | s_i]] - C(z_i), \quad (18)$$

where x_i is trader i 's optimal demand. In equilibrium every trader's attention choice is the best response to other traders' best responses. We confine our analysis to symmetric equilibria.

For each trader i , let

$$\zeta_i = \frac{\text{Var}(f)}{\text{Var}(\varepsilon_i)} = \frac{\frac{1}{\tau_v} + \frac{1}{\alpha}}{\frac{1}{\kappa z_i}}, \quad (19)$$

so that, in a symmetric equilibrium, the attention of any given trader is

$$z = \frac{\zeta}{\kappa} \frac{1}{\frac{1}{\tau_v} + \frac{1}{\alpha}}, \quad (20)$$

and thus, holding clarity and accuracy fixed, attention is one-to-one with ζ .

The quantity ζ is a signal-to-noise ratio, adapted to our model. We typically think of signal-to-noise ratios as the variation of information over the variation of noise contained in a signal. In our setting there are two noises in traders' signals, one common (ϕ) and one idiosyncratic (ε_i). Nevertheless we can interpret ζ as a signal-to-noise ratio by relating our context to that of senders' and receivers' noises of Myatt and Wallace (2012). In our context the sender is the firm, and it sends the signal $f = v + \phi$ to the traders, who receive it with additional processing noise ε_i . The traders cannot reduce the noise ϕ in the firm's signal; the best they can do is to try to reduce ε_i . In this sense, the relevant noise for them is ε_i and the relevant underlying "information" is the firm's signal f .

Lemma 6 *Other traders paying attention is a substitute for the i th trader paying attention,*

$$\left. \frac{d\zeta_i}{d\zeta_{-i}} \right|_{\zeta_{-i}=\zeta_i} < 0. \quad (21a)$$

and the substitutes become stronger with more traders,

$$\left. \frac{d}{dN} \left| \frac{d\zeta_i}{d\zeta_{-i}} \right| \right|_{\zeta_{-i}=\zeta_i} > 0. \quad (21b)$$

According to Lemma 6, a trader pays less attention to the disclosure as other traders pay more attention. This substitutability arises because higher aggregate attention makes the total order flow more informative for the market maker. In response to the market maker learning more, each trader scales back his attention so as to maximize his net utility by cutting down on costs. For the same reason, the substitutes become stronger with more traders, because in that case the order flow aggregates information more precisely.

We next examine how attention responds to accuracy and clarity.

Theorem 7 *There exists a unique symmetric equilibrium with endogenous attention z for large enough N . Attention increases in accuracy, it decreases in the number of traders, and it is hump-shaped in clarity.*

That the equilibrium is unique is a consequence of the substitutability in attention. The remaining three properties of attention have the following intuition. First, higher accuracy reduces uncertainty about the fundamental, which in turn increases the traders' incentives to pay attention to the disclosure. Second, the profit of an individual trader decreases with more competition, and this reduces his demand for costly attention. Third, when clarity is low, the net benefit of paying more attention is low relative to its cost. In contrast, when

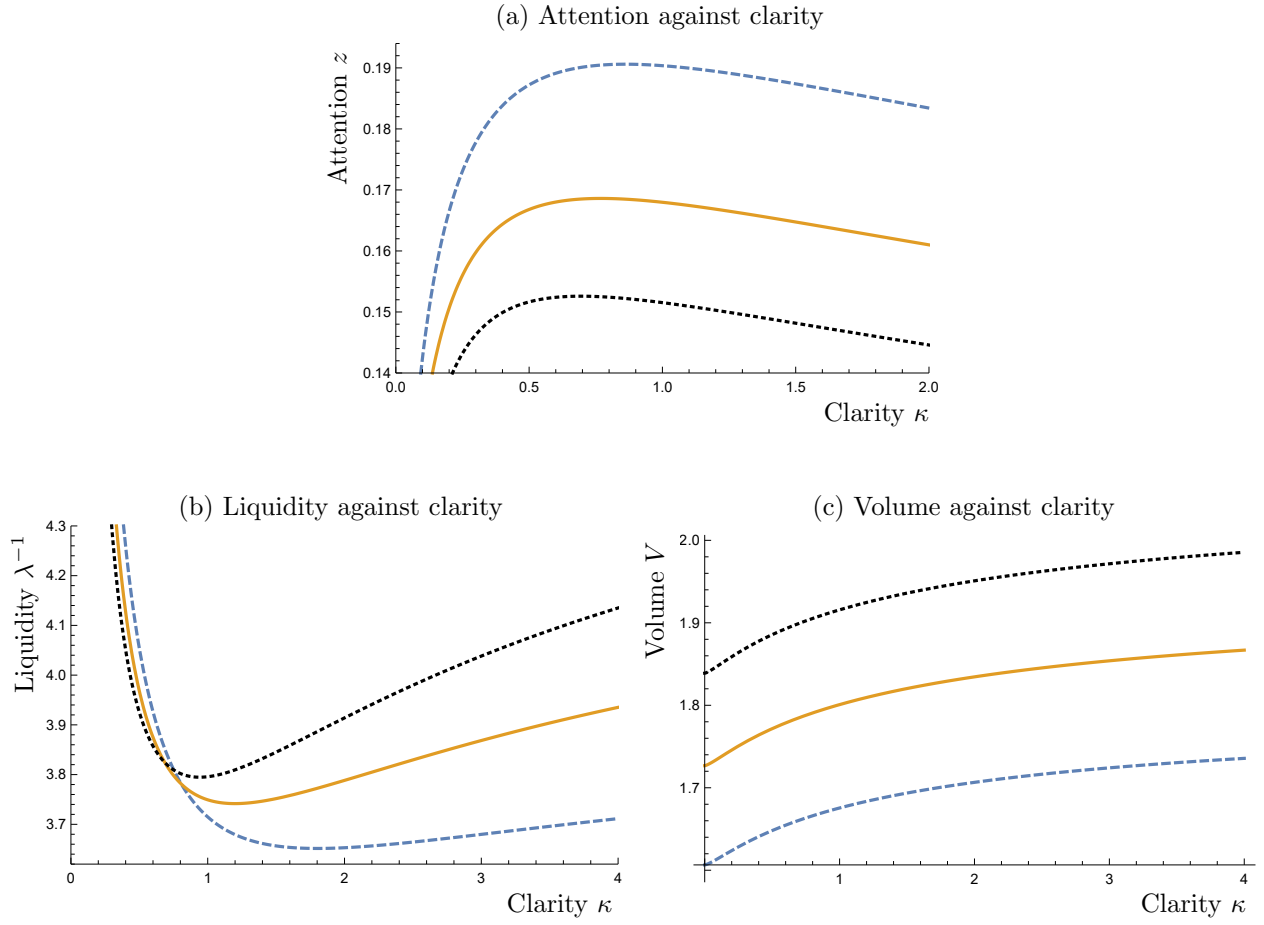


Figure 2: Equilibrium with endogenous attention for different numbers of traders. We show liquidity λ^{-1} , expected trading volume V , and attention as a function of disclosure clarity, for $N = 6$ (in dashed blue), $N = 8$ (in solid orange), and $N = 10$ (in dotted black). Our remaining parameters are $\alpha = 1$, $\tau_\psi = 1$, $\tau_v = 1$, and $c = 1$.

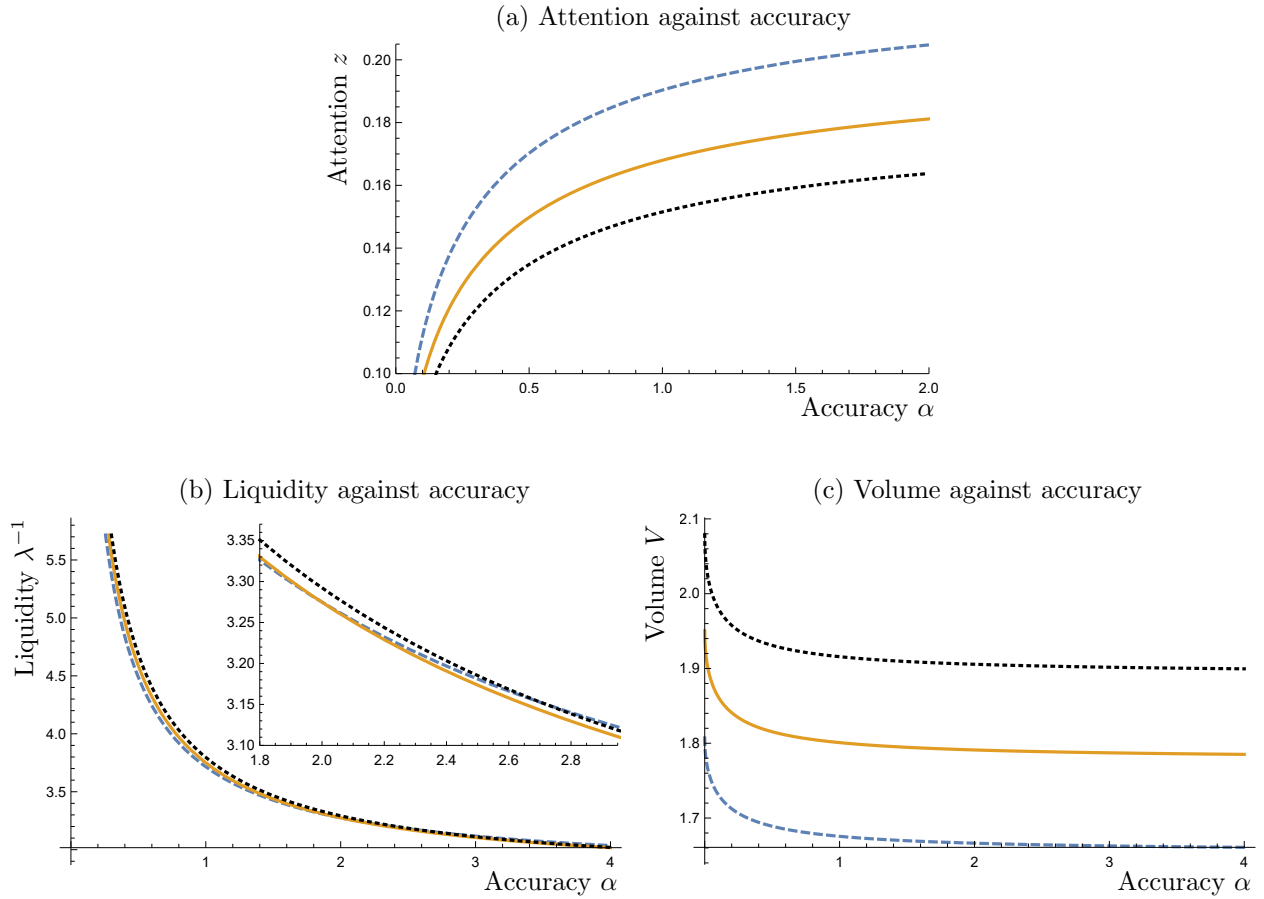


Figure 3: Equilibrium with endogenous attention for different numbers of traders. We show liquidity λ^{-1} , expected trading volume V , and attention as a function of disclosure accuracy, for $N = 6$ (in dashed blue), $N = 8$ (in solid orange), and $N = 10$ (in dotted black). Our remaining parameters are $\kappa = 1$, $\tau_\psi = 1$, $\tau_v = 1$, and $c = 1$.

clarity is high, the disclosure is very easy to understand, and it thus takes little effort to process. Consequently, trader attention is high for intermediate clarity, but it is lower for high and low clarity.¹²

We now revisit the properties of liquidity, volume and profit.

Theorem 8 *With endogenous symmetric attentions*

(i) *liquidity decreases in clarity for $N \leq 3$, but it is U-shaped in clarity for $N > 3$.*

Moreover, liquidity decreases in accuracy if accuracy is high enough.

(ii) *Volume increases in clarity, but it decreases in accuracy if $N > 1$.*

(iii) *Trader profit increases in accuracy. It increases in clarity for $N \leq 3$, but it is hump-shaped in clarity for $N > 3$.*

According to Theorem 8,—and as Figures 1, 2, and 3 confirm—in qualitative terms our comparative statics with exogenous attention continue to hold with endogenous attention. We must, however, point out that with endogenous attention our condition in the comparative static for accuracy becomes slightly weaker.¹³

3 Conclusion

We develop an economy with traders who absorb information partially. We study two dimensions of firm disclosure, accuracy and clarity, and show that they affect markets in economically different ways. First, disclosing more accurate information reduces liquidity.

¹²Our result that attention is hump-shaped in clarity is separate from how clarity affects liquidity. Market impact λ , the inverse of liquidity, is hump-shaped in clarity regardless of whether attention is endogenous.

¹³(A.109) of the Appendix shows a detailed sufficient condition.

In contrast, disclosing clearer information has a U-shaped effect on liquidity if competition among traders is sufficiently high. Second, trading volume decreases in accuracy, but increases in clarity. Third, trader attention to firm disclosures increases in accuracy, but is hump-shaped in clarity. Our results overall suggest that improving clarity is in the firm’s best interest as long as its existing clarity is high enough, and they are consistent with empirical evidence that lack of clarity may reduce liquidity and volume.

Our analysis reinforces recent concerns of firms and investors. According to KPMG (2011) and Monga and Chasan (2015), corporate disclosures are becoming longer and more complex to navigate. As further emphasised by SEC chairman Jay Clayton, “over the last two decades [...] the median word-count for SEC filings has more than doubled, yet the readability of those documents is at all-time low” (Clayton, 2017). It appears that while disclosure accuracy is increasing, disclosure clarity is decreasing. Our results suggest that this trend is detrimental to market liquidity and trading volume.

Our paper takes a first step in understanding the differential impact of disclosure accuracy and disclosure clarity on financial markets. Future work can explore how accuracy and clarity affect markets when traders have access to multiple sources of firm information, how traders may allocate their attention in an intertemporal manner, and how firms may use accuracy and clarity to manage their earnings.

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A Proofs

Proof of Lemma 1. From (9) and (4) we get

$$\lambda = \frac{Cov\left(v, \sum_{j=1}^N x_j + \psi\right)}{Var\left(\sum_{j=1}^N x_j + \psi\right)} = \frac{\frac{1}{\tau_v} \sum_{j=1}^N \beta_j}{\left(\frac{1}{\tau_v} + \frac{1}{\alpha}\right) \left(\sum_{j=1}^N \beta_j\right)^2 + \sum_{j=1}^N \frac{\beta_j^2}{\kappa z_j} + \frac{1}{\tau_\psi}}, \quad (\text{A.22})$$

which further implies that

$$\lambda^2 = \tau_\psi \left[\frac{1}{\tau_v} \sum_{j=1}^N \lambda \beta_j - \left(\frac{1}{\tau_v} + \frac{1}{\alpha}\right) \left(\sum_{j=1}^N \lambda \beta_j\right)^2 - \sum_{j=1}^N \frac{\lambda^2 \beta_j^2}{\kappa z_k} \right] \quad (\text{A.23})$$

Let

$$\tau_{s,i} = \frac{1}{Var(\phi + \varepsilon_i)} = \frac{\alpha \kappa z_i}{\alpha + \kappa z_i}. \quad (\text{A.24})$$

stand for the precision of the noise in the signal of informed trader i .

The profit of informed trader i is

$$\pi_i = E[x_i(v - p) | s_i] = x_i E \left[v - \lambda \left(\sum_{\substack{j=1 \\ j \neq i}}^N x_j + \psi \right) \middle| s_i \right] - \lambda x_i^2 \quad (\text{A.25})$$

The first order condition leads to the optimal demand

$$x_i = \frac{1}{2\lambda} E \left[v - \lambda \left(\sum_{\substack{j=1 \\ j \neq i}}^N x_j + \psi \right) \middle| s_i \right] = \frac{1}{2\lambda} \frac{\tau_{s_i}}{\tau_v + \tau_{s_i}} \left[1 - \left(1 + \frac{\tau_v}{\alpha} \right) \lambda \sum_{\substack{j=1 \\ k \neq i}}^N \beta_j \right] s_i \quad (\text{A.26})$$

Comparing the coefficient of s_i in (8) and (A.26), we have

$$\beta_i = \frac{1}{2\lambda} \frac{\tau_{s_i}}{\tau_v + \tau_{s_i}} \left[1 - \left(1 + \frac{\tau_v}{\alpha} \right) \lambda \sum_{\substack{j=1 \\ j \neq i}}^N \beta_j \right], \quad (\text{A.27})$$

and solving for $\lambda\beta_i$ we obtain

$$\lambda\beta_i = h_i \left[1 - \left(1 + \frac{\tau_v}{\alpha} \right) \sum_{j=1}^N \lambda\beta_j \right]. \quad (\text{A.28})$$

where

$$h_i = \frac{\tau_{s_i}}{2\tau_v + \tau_{s_i} \left(1 - \frac{\tau_v}{\alpha} \right)} = \frac{\alpha\kappa z_i}{(\tau_v + \alpha)\kappa z_i + 2\tau_v\alpha} = \frac{1}{1 + \frac{\tau_v}{\alpha} + 2\frac{\tau_v}{\kappa z_i}}. \quad (\text{A.29})$$

Summing (A.28) over i yields a solution for $\sum_{j=1}^N \lambda\beta_j$, which is

$$\sum_{j=1}^N \lambda\beta_j = \frac{H}{1 + \left(1 + \frac{\tau_v}{\alpha} \right) H}, \quad (\text{A.30})$$

where

$$H = \sum_{j=1}^N h_j. \quad (\text{A.31})$$

Substituting (A.30) back into (A.28) gives

$$\lambda\beta_i = \frac{h_i}{1 + \left(1 + \frac{\tau_v}{\alpha} \right) H}, \quad (\text{A.32})$$

which also gives

$$\sum_{j=1}^N \frac{(\lambda\beta_j)^2}{\kappa z_j} = \left(\frac{1}{1 + \left(1 + \frac{\tau_v}{\alpha} \right) H} \right)^2 \sum_{j=1}^N \frac{h_j^2}{\kappa z_j}. \quad (\text{A.33})$$

Substituting the value of (A.30) and (A.33) into (A.23) we obtain

$$\lambda^2 = \tau_\psi \left(\frac{1}{\tau_v} H - \sum_{j=1}^N \frac{h_j^2}{\kappa z_j} \right) \left(\frac{1}{1 + (1 + \frac{\tau_v}{\alpha}) H} \right)^2 \quad (\text{A.34})$$

From (A.30) we obtain

$$\lambda = \frac{H}{1 + (1 + \frac{\tau_v}{\alpha}) H} \frac{1}{\sum_{j=1}^N \beta_j}, \quad (\text{A.35})$$

which proves (10b). Squaring both sides of (A.35) and comparing with (A.34) we get

$$\left(\sum_{j=1}^N \beta_j \right)^2 = \frac{H^2}{\tau_\psi \left(\frac{1}{\tau_v} H - \sum_{j=1}^N \frac{h_j^2}{\kappa z_j} \right)} = \frac{1}{\tau_\psi \left(\frac{1}{\tau_v H} - \frac{\sum_{j=1}^N \frac{h_j^2}{\kappa z_j}}{H^2} \right)}. \quad (\text{A.36})$$

■

Lemma A.1 *In a symmetric equilibrium, the noise term is*

$$\nu = \frac{N+1}{N} \left(1 + \frac{\tau_v}{\alpha} \right) + \frac{2}{N} \frac{\tau_v}{\kappa z}, \quad (\text{A.37})$$

the aggregate trading intensity is given by

$$\left(\sum_{j=1}^N \beta_j \right)^2 = N \tau_\psi^{-1} \left(\frac{1}{\tau_v} + \frac{1}{\alpha} + \frac{1}{\kappa z} \right)^{-1}, \quad (\text{A.38})$$

and the market impact parameter is given by

$$\lambda^2 = \left(\frac{1}{1 + \frac{\tau_v}{\alpha} + \frac{1}{N} \left(1 + \frac{\tau_v}{\alpha} + 2 \frac{\tau_v}{\kappa z} \right)} \right)^2 \frac{\tau_\psi}{N} \left(\frac{1}{\tau_v} + \frac{1}{\alpha} + \frac{1}{\kappa z} \right). \quad (\text{A.39})$$

Proof. Let $z_1 = z_2 = \dots = z_N = z$ be fixed. Then

$$H = \frac{N}{1 + \frac{\tau_v}{\alpha} + 2\frac{\tau_v}{\kappa z}}, \quad (\text{A.40})$$

and thus the noise term is

$$\nu = 1 + \frac{\tau_v}{\alpha} + \frac{1}{H} = \frac{N+1}{N} \left(1 + \frac{\tau_v}{\alpha}\right) + \frac{2}{N} \frac{\tau_v}{\kappa z}, \quad (\text{A.41})$$

which proves (A.37). Due to symmetry we obtain

$$h_i^2 = \left(\frac{1}{1 + \frac{\tau_v}{\alpha} + 2\frac{\tau_v}{\kappa z}} \right)^2 \quad (\text{A.42})$$

for all i , which, by (A.36), implies that

$$\left(\sum_{j=1}^N \beta_j \right)^2 = \tau_\psi^{-1} \left(\frac{1}{\tau_v H} - \frac{\sum_{j=1}^N \frac{h_j^2}{\kappa z_j}}{H^2} \right)^{-1} = N \tau_\psi^{-1} \left(\frac{1}{\tau_v} + \frac{1}{\alpha} + \frac{1}{\kappa z} \right)^{-1}. \quad (\text{A.43})$$

This proves (A.38). Finally, squaring (10b) we get

$$\lambda^2 = \left(\frac{1}{1 + \frac{\tau_v}{\alpha} + \frac{1}{N} \left(1 + \frac{\tau_v}{\alpha} + 2\frac{\tau_v}{\kappa z}\right)} \right)^2 \frac{\tau_\psi}{N} \left(\frac{1}{\tau_v} + \frac{1}{\alpha} + \frac{1}{\kappa z} \right), \quad (\text{A.44})$$

which proves (A.39). ■

Lemma A.2 *In a symmetric equilibrium, the volume is*

$$\begin{aligned}
V &= \frac{1}{\sqrt{2\pi\tau_\psi}} \left[\sqrt{\frac{\tau_\psi}{\tau_v} \frac{\sum_{j=1}^N \beta_j}{\lambda}} + \sqrt{N+1} \right] \\
&= \frac{1}{\sqrt{2\pi\tau_\psi}} \left[\sqrt{\frac{(N+1) \left(\frac{1}{\tau_v} + \frac{1}{\alpha} \right) + 2\frac{1}{\kappa z}}{\frac{1}{\tau_v} + \frac{1}{\alpha} + \frac{1}{\kappa z}}} + \sqrt{N+1} \right]. \quad (\text{A.45})
\end{aligned}$$

Proof. By standard properties of Normal random variables, Equation (6) implies

$$\begin{aligned}
V_m &= \sqrt{\frac{2}{\pi}} \sqrt{\text{Var} \left(\sum_{j=1}^N x_j + \psi \right)} = \sqrt{\frac{2}{\pi}} \sqrt{\left(\frac{1}{\tau_v} + \frac{1}{\alpha} \right) \left(\sum_{j=1}^N \beta_j \right)^2 + \sum_{j=1}^N \frac{\beta_j^2}{\kappa z_j} + \frac{1}{\tau_\psi}} \\
&= \sqrt{\frac{2}{\pi}} \sqrt{\frac{1}{\tau_v} \frac{\sum_{j=1}^N \beta_j}{\lambda}}, \quad (\text{A.46})
\end{aligned}$$

where the last equality follows from (A.22). Similarly, Equation (7) implies

$$\begin{aligned}
V_c &= \sqrt{\frac{2}{\pi}} \left[\sum_{j=1}^N \sqrt{\text{Var}(x_j)} + \sqrt{\text{Var}(\psi)} \right] \\
&= \sqrt{\frac{2}{\pi}} \left[\sum_{j=1}^N \beta_j \sqrt{\frac{1}{\tau_v} + \frac{1}{\alpha} + \frac{1}{\kappa z_j} + \frac{1}{\sqrt{\tau_\psi}}} \right] = \sqrt{\frac{2}{\pi}} \left[\sqrt{\frac{1}{\tau_v} + \frac{1}{\alpha} + \frac{1}{\kappa z}} \sum_{j=1}^N \beta_j + \frac{1}{\sqrt{\tau_\psi}} \right] \\
&= \sqrt{\frac{2}{\pi}} \frac{\sqrt{N+1}}{\tau_\psi}, \quad (\text{A.47})
\end{aligned}$$

where the third equality follows by symmetry and the last equality follows by (A.43).

The first equality in (A.45) now follows by combining (A.46) and (A.47), and the second equality in (A.45) follows by using (A.35) and (A.40), and carrying out the algebra. ■

Proof of Theorem 2. We use the results in Lemma A.1 and Lemma A.2. The noise term

in (A.37) is

$$1 + \frac{\tau_v}{\alpha} + \frac{1}{H} = \frac{N+1}{N} \left(1 + \frac{\tau_v}{\alpha}\right) + \frac{2}{N} \frac{\tau_v}{\kappa z}, \quad (\text{A.48})$$

which is decreasing in κ . The square of the aggregate trading intensity in (A.38) is

$$\left(\sum_{j=1}^N \beta_j\right)^2 = N\tau_\psi^{-1} \left(\frac{1}{\tau_v} + \frac{1}{\alpha} + \frac{1}{\kappa z}\right)^{-1}, \quad (\text{A.49})$$

which is increasing in κ . It follows that the aggregate trading intensity is increasing in κ .

This proves claim (i).

Differentiating (A.39) with respect to κ we obtain

$$\frac{d\lambda^2}{d\kappa} = \frac{N\tau_\psi\alpha z}{[2\alpha\tau_v + (N+1)(\alpha + \tau_v)\kappa z]^3} [2\alpha\tau_v - (N-3)(\alpha + \tau_v)\kappa z], \quad (\text{A.50})$$

which is positive for

$$N < 3 + 2\frac{\alpha\tau_v}{\kappa z(\alpha + \tau_v)}, \quad (\text{A.51})$$

and negative for

$$N > 3 + 2\frac{\alpha\tau_v}{\kappa z(\alpha + \tau_v)}. \quad (\text{A.52})$$

If $N \leq 3$, (A.51) is always true, and thus $\frac{d\lambda^2}{d\kappa} > 0$. If, however, $N > 3$, $\frac{d\lambda^2}{d\kappa} > 0$ for

$$\kappa < \frac{2}{N-3} \frac{\alpha\tau_v}{z(\alpha + \tau_v)}. \quad (\text{A.53})$$

and $\frac{d\lambda^2}{d\kappa} < 0$ for

$$\kappa > \frac{2}{N-3} \frac{\alpha\tau_v}{z(\alpha + \tau_v)}. \quad (\text{A.54})$$

The above prove claim (ii).

Finally, differentiating (A.45) with respect to κ we obtain

$$\frac{dV}{d\kappa} = \frac{N-1}{2\sqrt{2\pi\tau_\psi}} \frac{z\left(\frac{1}{\tau_v} + \frac{1}{\alpha}\right)}{\left[1 + \kappa z\left(\frac{1}{\tau_v} + \frac{1}{\alpha}\right)\right]^2} \sqrt{\frac{\frac{1}{\tau_v} + \frac{1}{\alpha} + \frac{1}{\kappa z}}{(N+1)\left(\frac{1}{\tau_v} + \frac{1}{\alpha}\right) + 2\frac{1}{\kappa z}}}, \quad (\text{A.55})$$

which is positive. Claim (iii) thus holds. ■

Proof of Corollary 3. In the proof of claim (ii) of Theorem 2 we have the conditions for which increasing κ decreases or increases λ . In particular, conditions (A.53) and (A.54) prove that the clarity threshold is

$$\kappa_* = \frac{2}{(N-3)z} \frac{1}{\frac{1}{\tau_v} + \frac{1}{\alpha}}. \quad (\text{A.56})$$

By inspection it follows that κ_* decreases in N and increases in α . ■

Proof of Theorem 4. The proof follows a similar argument to that of Theorem 2 above. We again appeal to Lemma A.1 for the expressions for the noise term, the aggregate trading intensity, and the market impact parameter.

The noise term is

$$\frac{N+1}{N} \left(1 + \frac{\tau_v}{\alpha}\right) + \frac{2}{N} \frac{\tau_v}{\kappa z}, \quad (\text{A.57})$$

which is decreasing in α . The square of the aggregate trading intensity is

$$\left(\sum_{j=1}^N \beta_j\right)^2 = N\tau_\psi^{-1} \left(\frac{1}{\tau_v} + \frac{1}{\alpha} + \frac{1}{\kappa z}\right)^{-1}, \quad (\text{A.58})$$

which is increasing in α . Thus the aggregate trading intensity increases in α . Claim (i) follows.

Differentiating the expression for λ^2 with respect to α we obtain

$$\frac{d\lambda^2}{d\alpha} = \frac{N\tau_\psi\kappa^2z^2}{[2\alpha\tau_v + (N+1)(\alpha + \tau_v)\kappa z]^3} [2\alpha N\tau_v + (N+1)(\alpha + \tau_v)\kappa z], \quad (\text{A.59})$$

which is positive; this proves claim (ii).

Finally, differentiating (A.45) with respect to α we get

$$\frac{dV}{d\alpha} = -\frac{N-1}{2\sqrt{2\pi\tau_\psi}} \frac{z\kappa}{\alpha^2 \left[1 + \kappa z \left(\frac{1}{\tau_v} + \frac{1}{\alpha}\right)\right]^2} \sqrt{\frac{\frac{1}{\tau_v} + \frac{1}{\alpha} + \frac{1}{\kappa z}}{(N+1)\left(\frac{1}{\tau_v} + \frac{1}{\alpha}\right) + 2\frac{1}{\kappa z}}}, \quad (\text{A.60})$$

which is negative. Claim (iii) thus holds. ■

Lemma A.3 *The expected profit of trader i is*

$$E[\pi_i] = \left[\frac{1 + \frac{\tau_v}{\alpha} + \frac{\tau_v}{\kappa z_i}}{\left(1 + \frac{\tau_v}{\alpha} + 2\frac{\tau_v}{\kappa z_i}\right)^2} \right] \left[\tau_v \tau_\psi \sum_{j=1}^N \frac{1 + \frac{\tau_v}{\alpha} + \frac{\tau_v}{\kappa z_j}}{\left(1 + \frac{\tau_v}{\alpha} + 2\frac{\tau_v}{\kappa z_j}\right)^2} \right]^{-\frac{1}{2}} \left[1 + \left(1 + \frac{\tau_v}{\alpha}\right) \sum_{j=1}^N \frac{1}{1 + \frac{\tau_v}{\alpha} + 2\frac{\tau_v}{\kappa z_j}} \right]^{-1}. \quad (\text{A.61})$$

Proof. From (A.25) and (A.26) we get

$$\pi_i = 2\lambda x_i^2 - \lambda x_i^2 = \lambda x_i^2. \quad (\text{A.62})$$

Taking expectations we obtain

$$E[\pi_i] = \lambda\beta_i^2 E[s_i^2] = \lambda\beta_i^2 \left(\frac{1}{\tau_v} + \frac{1}{\alpha} + \frac{1}{\kappa z_i} \right). \quad (\text{A.63})$$

Combining (A.32), (A.34) and (A.63) we get (A.61). ■

Proof of Lemma 5. Setting $z_i = z$ for all i in Lemma A.3 yields

$$E[\pi_i] = \sqrt{\frac{N^3}{\tau_\psi \tau_v^2} \frac{\sqrt{\kappa z \left[1 + \kappa z \left(\frac{1}{\tau_v} + \frac{1}{\alpha}\right)\right]}}{2 + (N+1)\kappa z \left(\frac{1}{\tau_v} + \frac{1}{\alpha}\right)}}. \quad (\text{A.64})$$

Differentiating the profit with respect to α we obtain

$$\frac{d}{d\alpha} E[\pi_i] = \frac{N^2 \kappa^2 z^2 \sqrt{\tau_v}}{2\sqrt{N\tau_\psi \alpha \kappa z} [\alpha \tau_v + (\alpha + \tau_v) \kappa z]} \frac{[2\alpha N \tau_v + (N+1)(\alpha + \tau_v) \kappa z]^2}{[2\alpha \tau_v + (N+1)(\alpha + \tau_v) \kappa z]^3}, \quad (\text{A.65})$$

which is positive. Differentiating the profit with respect to κ we obtain

$$\begin{aligned} \frac{d}{d\kappa} E[\pi_i] = & \frac{N^2 \alpha^2 z \sqrt{\tau_v}}{2\sqrt{N\tau_\psi \alpha \kappa z} [\alpha \tau_v + (\alpha + \tau_v) \kappa z]} \frac{[2\alpha N \tau_v + (N+1)(\alpha + \tau_v) \kappa z]}{[2\alpha \tau_v + (N+1)(\alpha + \tau_v) \kappa z]^3} \\ & [2\alpha \tau_v - (N-3)(\alpha + \tau_v) \kappa z], \quad (\text{A.66}) \end{aligned}$$

which is positive for

$$N < 3 + 2 \frac{\alpha \tau_v}{\kappa z (\alpha + \tau_v)}, \quad (\text{A.67})$$

and negative for

$$N > 3 + 2 \frac{\alpha \tau_v}{\kappa z (\alpha + \tau_v)}. \quad (\text{A.68})$$

If $N \leq 3$, (A.67) is always true, and thus $dE[\pi_i]/d\kappa > 0$. If, however, $N > 3$, $dE[\pi_i]/d\kappa > 0$ for

$$\kappa < \frac{2}{N-3} \frac{\alpha \tau_v}{z(\alpha + \tau_v)}. \quad (\text{A.69})$$

and $dE[\pi_i]/d\kappa < 0$ for

$$\kappa > \frac{2}{N-3} \frac{\alpha\tau_v}{z(\alpha + \tau_v)}. \quad (\text{A.70})$$

■

Proof of Lemma 6. The first-order condition of trader i is

$$\frac{d}{dz_i} E[\pi_i] - C'(z_i) = 0. \quad (\text{A.71})$$

Let ζ_i for all i be as in (19). Taking the first-order condition of a particular trader i in (A.71) while holding the attention of all other traders at z_{-i} , and rewriting the first-order condition in ζ_i and ζ_{-i} shows, by Lemma A.3 and after some algebra, that

$$\begin{aligned} & \frac{\left(\frac{1}{\tau_v} + \frac{1}{\alpha}\right)^3 \kappa^4}{4c^2\tau_v^2\tau_\psi} (\zeta_{-i} + 2)^2 \{\zeta_i [(N+1)\zeta_{-i} + 4] + 2N\zeta_{-i} + 4\}^{-4} \\ & \left\{ \zeta_i^2 [N\zeta_{-i}^2 + (N+3)\zeta_{-i} + 4] + \zeta_i [(4N-3)\zeta_{-i}^2 + 4N\zeta_{-i} + 4] + 4(N-1)\zeta_{-i}(\zeta_{-i} + 1) \right\}^{-3} \\ & \quad \left\{ \zeta_i^3 [(6N^2 - N - 3)\zeta_{-i}^3 + (6N^2 + 24N - 18)\zeta_{-i}^2 + (28N - 4)\zeta_{-i} + 16] \right. \\ & \quad + \zeta_i^2 [(28N^2 - 19N - 5)\zeta_{-i}^3 + (28N^2 + 60N - 64)\zeta_{-i}^2 + (84N - 36)\zeta_{-i} + 32] \\ & \quad \left. + 2\zeta_i(N\zeta_{-i} + 2) [(20N - 19)\zeta_{-i}^2 + (20N - 16)\zeta_{-i} + 4] \right. \\ & \quad \left. + 16(N-1)\zeta_{-i}(\zeta_{-i} + 1)(\zeta_{-i} + 2) \right\}^2 - \zeta_i^2 = 0 \quad (\text{A.72}) \end{aligned}$$

Equation (A.72) determines how ζ_i responds to ζ_{-i} . Let $J(\zeta_i, \zeta_{-i})$ denote its left-hand side.

Applying the Implicit Function Theorem we get

$$\frac{d\zeta_i}{d\zeta_{-i}} = -\frac{\frac{\partial}{\partial \zeta_{-i}} J}{\frac{\partial}{\partial \zeta_i} J} \quad (\text{A.73})$$

Evaluating (A.73) in equilibrium by setting $\zeta_{-i} = \zeta_i = \zeta$ we obtain

$$\left. \frac{d\zeta_i}{d\zeta_{-i}} \right|_{\zeta_{-i}=\zeta_i} = -(N-1) \frac{L_1(\zeta)}{L_2(\zeta)} \quad (\text{A.74})$$

where

$$\begin{aligned} L_1 = & \zeta^4 (42N^3 - 23N^2 - 60N - 27) + \zeta^3 (64N^3 + 68N^2 - 172N - 144) \\ & + \zeta^2 (24N^3 + 148N^2 - 88N - 264) + \zeta (64N^2 + 48N - 192) \\ & + 32N - 48, \quad (\text{A.75}) \end{aligned}$$

and

$$\begin{aligned} L_2 = & \zeta^5 (36N^4 + 30N^3 - 24N^2 - 18N) + \zeta^4 (68N^4 + 266N^3 - 3N^2 - 148N - 27) \\ & + \zeta^3 (48N^4 + 416N^3 + 372N^2 - 308N - 144) + \zeta^2 (16N^4 + 248N^3 + 596N^2 - 152N - 264) \\ & + \zeta (64N^3 + 320N^2 + 48N - 192) + 64N^2 + 32N - 48 \quad (\text{A.76}) \end{aligned}$$

By inspection, the coefficients of $L_1(\zeta)$ and $L_2(\zeta)$ are positive for $N > 1$, and thus $L_1(\zeta)$ and $L_2(\zeta)$ are both positive. This proves that

$$\left. \frac{d\zeta_i}{d\zeta_{-i}} \right|_{\zeta_{-i}=\zeta_i} < 0. \quad (\text{A.77})$$

A similar argument shows that the substitutes become stronger in N . Taking the derivative of the absolute value of $d\zeta_i/d\zeta_{-i}$ evaluated at $\zeta_{-i} = \zeta_i = \zeta$ (the quantity in (A.74) with the sign flipped) with respect to N gives an expression that is the ratio of two polynomials of ζ ,

both of which are positive for $N > 1$. ■

Proposition A.4 *A symmetric equilibrium with endogenous attention is characterized by*

$$F(\zeta; \kappa, \alpha) \equiv \frac{4\tau_\psi c^2 \tau_v^2}{\kappa^4 \left(\frac{1}{\tau_v} + \frac{1}{\alpha}\right)^3} N^3 \zeta^3 (\zeta + 1) (\zeta + 2)^2 [\zeta(N + 1) + 2]^4 - [\zeta^2 (6N^2 - N - 3) + 2\zeta (2N^2 + 5N - 4) + 4(2N - 1)]^2 = 0, \quad (\text{A.78})$$

where

$$\zeta = \kappa z \left(\frac{1}{\tau_v} + \frac{1}{\alpha} \right). \quad (\text{A.79})$$

Such an equilibrium exists, and it is unique.

Proof. Setting $\zeta_i = \zeta_{-i} = \zeta$ in Equation (A.72) and rearranging establishes (A.78).

For fixed κ and α , $F(\zeta; \kappa, \alpha)$ in (A.78) is a polynomial of order 10 in ζ . The coefficients of orders five and higher are all positive. The coefficients of orders zero to four are in the following table.

term	coefficient
ζ^0	$-16(2N - 1)^2$
ζ^1	$-16(2N - 1)(2N^2 + 5N - 4)$
ζ^2	$-4(4N^4 + 44N^3 - 7N^2 - 50N + 22)$
ζ^3	$-4(2N^2 + 5N - 4)(6N^2 - N - 3) + 256 \frac{\tau_\psi c^2 \tau_v^2}{\kappa^4 \left(\frac{1}{\tau_v} + \frac{1}{\alpha}\right)^3} N^3$
ζ^4	$-(6N^2 - N - 3)^2 + 512 \frac{\tau_\psi c^2 \tau_v^2}{\kappa^4 \left(\frac{1}{\tau_v} + \frac{1}{\alpha}\right)^3} N^3 (N + 2)$

By inspection of the above table it follows that for large enough N , the coefficients of orders zero through three are negative. The sign of the coefficient of order four depends on the model parameters, even if N is large enough. Nevertheless, the signs of the coefficients of orders five through ten are all positive, which implies that there is either a sign switch between the coefficients of order three and four, or between the coefficients of order four and five. In either case there is exactly one sign switch among the coefficients of ζ in F in ascending order. It follows by Descartes' rule of signs that for large enough N there is a unique positive root for ζ . This proves that for large enough N there is a unique symmetric equilibrium in z , the solution for which is given as

$$z = \frac{\zeta}{\kappa \left(\frac{1}{\tau_v} + \frac{1}{\alpha}\right)}, \quad (\text{A.80})$$

where ζ is the unique positive root of (A.78). ■

Remark A.5 *In a symmetric equilibrium, the expected profit of each trader is*

$$E[\pi_i] = \frac{\lambda}{N\tau_\psi}. \quad (\text{A.81})$$

This follows from substituting (A.38) into (A.63) and using $z_i = z$ for all i and $(\sum_{j=1}^N \beta_j)^2 = \beta_i = \beta$ for all i .

Lemma A.6 *In the symmetric equilibrium with endogenous attention and $N > 3$, the following three inequalities are equivalent:*

(a)

$$2\alpha\tau_v - (N - 3)(\alpha + \tau_v)\kappa z > 0 \quad (\text{A.82})$$

(b)

$$\zeta < \frac{2}{N - 3} \quad (\text{A.83})$$

(c)

$$128 \frac{c^2 \tau_v^2 \tau_\psi}{\kappa^4 \left(\frac{1}{\tau_v} + \frac{1}{\alpha}\right)^3} > \frac{(N - 3)^6}{(N - 2)^2 (N - 1) N} \quad (\text{A.84})$$

Proof. The inequalities in (a) and (b) are equivalent by definition of ζ and by $N > 3$. Using the equilibrium condition in (A.78) provides a sign for $F\left(\frac{2}{N-3}; \kappa, \alpha\right)$, which, after some algebra, shows that (c) is equivalent to (b). ■

Lemma A.7 *In the symmetric equilibrium with endogenous attention*

$$\frac{d\zeta}{d\kappa} > 0 \quad (\text{A.85})$$

and

$$\frac{d\zeta}{d\alpha} < 0. \quad (\text{A.86})$$

Proof. Applying the Implicit Function Theorem on (A.78) gives

$$\frac{d\zeta}{d\kappa} = -\frac{\frac{\partial}{\partial \kappa} F}{\frac{\partial}{\partial \zeta} F} \quad (\text{A.87})$$

In equilibrium, we have

$$\frac{\partial}{\partial \zeta} F > 0. \quad (\text{A.88})$$

This is because $F(\zeta; \kappa, \alpha)$ is a polynomial in ζ with a positive leading coefficient, and because it has a unique positive root for ζ . These facts imply that $F(\zeta; \kappa, \alpha)$ crosses zero from below as ζ increases, and thus the slope of F is positive at the root for ζ . We also have

$$\frac{\partial}{\partial \kappa} F = -16 \frac{\tau_\psi c^2 \tau_v^2}{\kappa^5 \left(\frac{1}{\tau_v} + \frac{1}{\alpha}\right)^3} N^3 \zeta^3 (\zeta + 1) (\zeta + 2)^2 [\zeta(N + 1) + 2]^4 < 0, \quad (\text{A.89})$$

and thus

$$\frac{d\zeta}{d\kappa} > 0. \quad (\text{A.90})$$

Applying the Implicit Function Theorem on (A.78), but this time with respect to accuracy, we get

$$\frac{d\zeta}{d\alpha} = -\frac{\frac{\partial}{\partial \alpha} F}{\frac{\partial}{\partial \zeta} F}. \quad (\text{A.91})$$

We also have

$$\frac{\partial}{\partial \alpha} F = \frac{12\tau_\psi c^2 \tau_v^2}{\alpha^2 \kappa^4 \left(\frac{1}{\tau_v} + \frac{1}{\alpha}\right)^4} N^3 \zeta^3 (\zeta + 1) (\zeta + 2)^2 [\zeta(N + 1) + 2]^4 > 0, \quad (\text{A.92})$$

and thus, by (A.92), we obtain

$$\frac{d\zeta}{d\alpha} < 0. \quad (\text{A.93})$$

■

Proof of Theorem 7. Existence and uniqueness follow immediately from Proposition A.4.

By the chain rule and by (A.87) we get

$$\frac{dz}{d\kappa} = \frac{d(z\kappa\frac{1}{\kappa})}{d\kappa} = \frac{1}{\kappa^2} \left(\kappa \frac{d(z\kappa)}{d\kappa} - z\kappa \right) = -\frac{1}{\kappa^2 \left(\frac{1}{\tau_v} + \frac{1}{\alpha} \right)} \frac{\kappa \frac{\partial}{\partial \kappa} F + \zeta \frac{\partial}{\partial \zeta} F}{\frac{\partial}{\partial \zeta} F} \quad (\text{A.94})$$

From above we know that $\frac{\partial}{\partial \zeta} F > 0$, and thus the denominator of the second fraction above is positive. The sign of $\frac{dz}{d\kappa}$ therefore boils down to the sign of the numerator of the second fractional term above. We prove that $\kappa \frac{\partial}{\partial \kappa} F + \zeta \frac{\partial}{\partial \zeta} F$ is negative for small κ , positive for large κ , and it crosses zero only once, which implies that z is a hump-shaped function of κ . We have

$$\begin{aligned} \kappa \frac{\partial}{\partial \kappa} F(\zeta; \kappa, \alpha) + \zeta \frac{\partial}{\partial \zeta} F(\zeta; \kappa, \alpha) = \\ \frac{4\tau_\psi c^2 \tau_v^2}{\kappa^4 \left(\frac{1}{\tau_v} + \frac{1}{\alpha} \right)^3} \frac{N^3 \zeta^3 (\zeta + 2) [\zeta(N + 1) + 2]^3}{\zeta^2 (6N^2 - N - 3) + 2\zeta (2N^2 + 5N - 4) + 4(2N - 1)} G(\zeta) \end{aligned} \quad (\text{A.95})$$

where $G(\zeta)$ is a fifth-order polynomial in ζ that does not depend on κ or α . Using the

equilibrium condition in (A.78) we can simplify the above to

$$\kappa \frac{\partial}{\partial \kappa} F(\zeta; \kappa, \alpha) + \zeta \frac{\partial}{\partial \zeta} F(\zeta; \kappa, \alpha) = \frac{\zeta^2 (6N^2 - N - 3) + 2\zeta (2N^2 + 5N - 4) + 4(2N - 1)}{(\zeta + 1)(\zeta + 2)[\zeta(N + 1) + 2]} G(\zeta) \quad (\text{A.96})$$

The first fraction in (A.96) is always positive, while the second fraction in (A.96) is positive by inspection because $N \geq 1$. The coefficients of $G(\zeta)$ are in the following table.

term	coefficient
ζ^0	$-16(2N - 1)$
ζ^1	$-16(5N - 4)$
ζ^2	$4(2N^3 - 7N^2 + 2N + 14)$
ζ^3	$4(4N^3 + N^2 + 17N - 2)$
ζ^4	$(22N^3 + 37N^2 + 8N - 23)$
ζ^5	$2(N + 1)(6N^2 - N - 3)$

By inspection of the above table it follows that for large enough N the coefficients of $G(\zeta)$ switch signs exactly once, between the linear and the quadratic terms. Thus, by Descartes' rule of signs, $G(\zeta)$ has a unique positive root in ζ . Moreover, because the leading coefficient of $G(\zeta)$ is positive, $G(\zeta)$ is positive for large and positive ζ , which, because there is only one positive root, also implies that $G(\zeta)$ crosses zero at its root from below. Thus $G(\zeta)$ is negative to the left of its root, and positive to the right of its root. Finally, because ζ is

one-to-one with κ , the above also imply that $G(\zeta)$ is negative for small κ and positive for large κ , and becomes zero as a function of κ exactly once. More concretely, let ζ_G be the root of $G(\zeta)$. We can support an equilibrium with for any ζ of our choice by setting κ to the value that corresponds to our choice for ζ . Solving (A.78) we obtain

$$\kappa = \left(\frac{4\tau_\psi c^2 \tau_v^2}{\left(\frac{1}{\tau_v} + \frac{1}{\alpha}\right)^3} \frac{N^3 \zeta^3 (\zeta + 1) (\zeta + 2)^2 [\zeta(N + 1) + 2]^4}{[\zeta^2 (6N^2 - N - 3) + 2\zeta (2N^2 + 5N - 4) + 4(2N - 1)]^2} \right)^{\frac{1}{4}} \quad (\text{A.97})$$

Evaluating (A.97) at $\zeta = \zeta_G$ gives the value of κ for which $G(\zeta) = 0$; let κ_G denote that value. Any κ to the left of κ_G yields a negative $G(\zeta)$, and any κ to the right of κ_G yields a positive $G(\zeta)$. The claim that z is hump-shaped in κ now follows by Equation (A.94).

Next, by the product rule we get

$$\frac{d(\kappa z)}{d\alpha} = \left(\frac{1}{\tau_v} + \frac{1}{\alpha}\right)^{-2} \left[\left(\frac{1}{\tau_v} + \frac{1}{\alpha}\right) \frac{d\zeta}{d\alpha} + \frac{1}{\alpha^2} \zeta \right] \quad (\text{A.98})$$

After using (A.91) and the equilibrium condition to simplify the algebra, the term in the second bracket reduces to

$$\left(\frac{1}{\tau_v} + \frac{1}{\alpha}\right) \frac{d\zeta}{d\alpha} + \frac{1}{\alpha^2} \zeta = \frac{\zeta^2 K_1(\zeta)}{\alpha^2 K_2(\zeta)} \quad (\text{A.99})$$

where $K_1(\zeta)$ is a fourth-order polynomial and $K_2(\zeta)$ is a fifth-order polynomial, with coefficients as in the following table.

term	coefficient in $K_1(\zeta)$	coefficient in $K_2(\zeta)$
ζ^0	$16N(2N + 1)$	$48(2N - 1)$
ζ^1	$8(2N^3 + 9N^2 + 12N - 5)$	$16(8N^2 + 19N - 12)$
ζ^2	$4(10N^3 + 27N^2 + 21N - 19)$	$4(10N^3 + 93N^2 + 90N - 82)$
ζ^3	$2(22N^3 + 39N^2 - 2N - 23)$	$4(28N^3 + 105N^2 + 33N - 70)$
ζ^4	$3(N + 1)(6N^2 - N - 3)$	$110N^3 + 201N^2 - 40N - 115$
ζ^5	0	$6(N + 1)(6N^2 - N - 3)$

By inspection it follows that the coefficients of $K_1(\zeta)$ and $K_2(\zeta)$ are all positive because $N \geq 1$. Because $\zeta > 0$, this further implies that $K_1(\zeta)/K_2(\zeta) > 0$. It now follows that

$$\frac{d(\kappa z)}{d\alpha} > 0, \tag{A.100}$$

and that

$$\frac{dz}{d\alpha} = \frac{1}{\kappa} \frac{d(\kappa z)}{d\alpha} > 0, \tag{A.101}$$

which proves that attention increases in accuracy.

Finally, we derive the sign of $d\zeta/dN$. By the Implicit Function Theorem on (A.78), we get

$$\frac{d\zeta}{dN} = -\frac{\frac{\partial}{\partial N} F}{\frac{\partial}{\partial \zeta} F}. \tag{A.102}$$

As above, $\partial F/\partial \zeta > 0$ in equilibrium. After using the equilibrium condition to simplify the

algebra, the numerator above becomes

$$\frac{\partial}{\partial N} F(\zeta; \kappa, \alpha) = \frac{\zeta^2 (6N^2 - N - 3) + 2\zeta (2N^2 + 5N - 4) + 4(2N - 1)}{N [\zeta(N + 1) + 2]} H(\zeta) \quad (\text{A.103})$$

where $H(\zeta)$ is a third-order polynomial, with coefficients as in the following table.

term	coefficient
ζ^0	$8(2N - 3)$
ζ^1	$2(16N^2 - 30)$
ζ^2	$2(6N^3 + 17N^2 - 24N - 21)$
ζ^3	$18N^3 - 11N^2 - 22N - 9$

It follows by inspection that the coefficients of $H(\zeta)$ are all positive for $N > 1$. Because $\zeta > 0$, this proves that $d\zeta/dN < 0$ for $N > 1$, and concludes the proof. ■

Proof of Theorem 8. By the chain rule and by differentiating (A.44) with respect to κz we obtain

$$\begin{aligned} \frac{d\lambda^2}{d\kappa} &= \frac{d\lambda^2}{d(\kappa z)} \frac{d(\kappa z)}{d\kappa} \\ &= \frac{N\tau_\psi \alpha^2 z}{[2\alpha\tau_v + (N + 1)(\alpha + \tau_v)\kappa z]^3} [2\alpha\tau_v - (N - 3)(\alpha + \tau_v)\kappa z] \frac{d(\kappa z)}{d\kappa}. \end{aligned} \quad (\text{A.104})$$

The first term on the right-hand side of (A.104) is positive by inspection. The second term on the right-hand side of (A.104) is always positive if $N \leq 3$, but if $N > 3$, then by Lemma A.6 it is positive for small κ , negative for large κ , and it crosses zero only once. The third

term on the right-hand side of (A.104) is positive because by the chain rule

$$\frac{d(\kappa z)}{d\kappa} = \frac{d(\kappa z)}{d\zeta} \frac{d\zeta}{d\kappa} = \left(\frac{1}{\tau_v} + \frac{1}{\alpha} \right)^{-1} \frac{d\zeta}{d\kappa} > 0. \quad (\text{A.105})$$

This proves that λ , which has the same monotonicity in κ as λ^2 , is hump-shaped in κ if $N > 3$. The first part of claim (i) follows.

Taking the total derivative of λ^2 with respect to α we have

$$\frac{d\lambda^2}{d\alpha} = \frac{\partial\lambda^2}{\partial\alpha} + \frac{\partial\lambda^2}{\partial(\kappa z)} \frac{d(\kappa z)}{d\alpha} \quad (\text{A.106})$$

By Theorem 2 it follows that $\partial\lambda^2/\partial\alpha > 0$. This, together with relation (A.100), implies that $d\lambda^2/d\alpha > 0$ as long as $\partial\lambda^2/\partial(\kappa z) > 0$. By (A.39) of Lemma A.1 we obtain

$$\frac{\partial\lambda^2}{\partial(\kappa z)} = \frac{N\tau_\psi\alpha^2 z}{[2\alpha\tau_v + (N+1)(\alpha + \tau_v)\kappa z]^3} [2\alpha\tau_v - (N-3)(\alpha + \tau_v)\kappa z] \quad (\text{A.107})$$

If $N \leq 3$ the above is always positive. If $N > 3$, by Lemma A.6 we have that $\partial\lambda^2/\partial(\kappa z) > 0$ if

$$128 \frac{c^2\tau_v^2\tau_\psi}{\kappa^4 \left(\frac{1}{\tau_v} + \frac{1}{\alpha} \right)^3} > \frac{(N-3)^6}{(N-2)^2(N-1)N} \quad (\text{A.108})$$

holds, or, equivalently, if

$$\alpha > \tau_v \left[4 \sqrt[3]{2 \frac{c^2\tau_v^5\tau_\psi}{\kappa^4} \frac{(N-2)^2(N-1)N}{(N-3)^6}} - 1 \right]^{-1} \quad (\text{A.109})$$

holds. The above prove the second part of claim (i) as stated.

By Lemma A.2 we can write the volume as

$$V = \frac{1}{\sqrt{2\pi\tau_\psi}} \left[\sqrt{\frac{(N+1)\zeta+2}{\zeta+1}} + \sqrt{N+1} \right], \quad (\text{A.110})$$

and thus, because V does not depend on κ other than through ζ ,

$$\frac{dV}{d\zeta} = \frac{1}{2\sqrt{2\pi\tau_\psi}} \sqrt{\frac{\zeta+1}{(N+1)\zeta+2}} \frac{(N-1)}{(\zeta+1)^2}, \quad (\text{A.111})$$

which is positive for $N > 1$. By the chain rule we then obtain

$$\frac{dV}{d\kappa} = \frac{dV}{d\zeta} \frac{d\zeta}{d\kappa}, \quad (\text{A.112})$$

which is positive, if $N > 1$, due to (A.85) of Lemma A.7. Moreover, V does not depend on α other than through ζ , and thus by the chain rule we obtain that $dV/d\alpha = (dV/d\zeta)(d\zeta/d\alpha)$, which is negative for $N > 1$ by (A.111) and (A.86) of Lemma A.7. Claim (ii) now follows.

The proof of claim (iii) follows by a similar argument. ■