# Risk seekers: trade, noise, and the rationalizing effect of market impact on convex preferences*

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**Abstract**

Long-held intuition dictates that trading among rational agents is impossible (Milgrom and Stokey, 1982; Tirole, 1982). Risk seekers can resolve this conundrum. Not only do such traders undertake gambles that risk averters prefer to avoid, but they also inject enough noise into prices to obscure everyone's private information. Markets are therefore inefficient strictly due to noise in traders' endogenous signals. Moreover, risk seekers act as utility maximizers because, unlike noise traders, they fully internalize their impact on prices. This behavior also implies that economies with even a few risk seekers are empirically distinct from noise-trading models of inefficient markets.

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# 1 Introduction

According to classic results on trade, it is impossible to have trade among fully rational economic agents. This conundrum poses a challenge to the field of Finance, because it suggests that any internally consistent theory of inefficient markets is inconsistent with information-based trade. It suggests that we must resort—as a long literature has done—to exogenous devices such as liquidity trading, random endowments, and behavioral effects.

It is, however, not only possible to have information-based trade, but to also think of prices as aggregators of noisy signals acquired by rational traders endogenously. To build such a theory, however, we must relax our assumptions on traders' attitudes towards risk. We can, more specifically, assume that at least some traders like risk. Such risk seekers are nevertheless fully rational, and their presence is necessary, because we cannot have equilibrium if everyone is risk neutral or risk averse (Milgrom and Stokey, 1982; Tirole, 1982). In short, we do not need to redefine rationality; we need only expand what we currently categorize as suitable preferences.

It may appear that risk seeking is a troublesome ingredient of any rational foundation of inefficient markets. After all, intuition from price-taking models dictates that with negative risk aversion the traders' demand-choice problem is not well posed. There are, however, several reasons why risk seeking is not only rational, but also economically salient.

First, we can motivate risk seeking behavior on two fronts. We can appeal to risk seeking as an innate preference. As I discuss in the following section, several papers in the empirical and experimental literature suggest that a minority fraction of people are risk seekers. Alternatively, we can think of negative risk aversion as a reduced-form model for an institutional trader whose reward depends on taking risk due to contracts with client investors.

Second, the risk seekers' demand choice is well posed indeed. Assuming, as in Kyle

(1989), that traders have mean-variance preferences, their second-order condition is the sum of their market impact and of a risk-preference term, so that—if market impact is large enough—preferences are concave in equilibrium even with negative risk aversion. In fact, the more these risk seekers like risk, the more aggressively they trade, and the more market impact they have. Their trading aggression thus concavifies their preferences, acting as a self-sustaining force both of their own rationality and of the overall equilibrium.

From a perspective of market inefficiency, having risk seekers in an economy also ensures that there is enough noise. Their trading aggression amplifies the signal noise they inject into prices, enabling risk averters to trade conservatively without precluding equilibrium. Due to a substitution effect in trading intensity that I discuss below, prices are noisier *in equilibrium* when some traders like risk. As a result, even small numbers of risk seekers can sustain enough noise to obscure fundamentals for everyone else; without them, prices reveal so much dividend information that no rational individual is willing to trade. To borrow a phrase from Dow and Gorton (1997)—who, among others, argue that noise is necessary for trade—there is "noise trade." Restated from a trading perspective, risk seekers enable trade: they take on the risk that risk averters would like to avoid.

To focus the discussion on the traders, I restrict the model to one trading period and one risky asset. Opting for simplicity of exposition, I adopt a market structure similar to Kyle (1985), albeit without noise traders: there is a risk-neutral market maker and many strategic traders with mean-variance preferences who submit orders without observing prices. This type of market, known in the literature as a "market-order" model, allows us to abstract away from effects of learning from prices. This happens without loss of generality for the main message of this paper—as I show with extensions, the equilibrium with traders who do observe prices exists if and only if one exists with traders who do not.[1]

[1]Beyond showing that we do not lose intuition if we exclude learning from prices, these extensions also

I present the main intuition using a toy model with homogeneous risk seekers. Setting the empirical plausibility of such preferences briefly aside, this model serves two purposes. First, we can elucidate that market impact is directly related to how much the traders like risk—this relationship becomes more opaque once we extend the model to heterogeneous risk preferences. Second, under information acquisition, we can empirically distinguish the economy with risk seekers from the canonical economy with noise traders: when information becomes cheaper, liquidity always increases in the economy with risk seekers, whereas in the one with noise traders it may decrease. As I discuss in more detail below, this distinction arises because if all traders are rational, then they internalize their impact on prices completely. No such thing happens in an economy where some traders are irrational.

But what if some traders are rational, yet not risk-seeking? Do we still have equilibrium then? To address these questions we must use an economy in which risk seeking coexists with other risk preferences. Nevertheless, by incorporating heterogeneous preferences into a standard economy we lose the tractability required for an answer with enough generality. Since the only thing required to maintain the spirit of the homogeneous model is a residual degree of market power, I appeal to a version of the economy with monopolistic competition based on Kyle (1989) adapting it to exclude noise traders. The resulting model treats the market as a large collection of small traders with residual market impact, allowing for full heterogeneity of risk preferences and even for information acquisition.[2]

I characterize all possible equilibria in this model. Generalizing the results from the

show that risk seeking can generate incentives to acquire private information. This suggests that the result of Grossman (1976)—that traders have no use for their private information because it is already incorporated in the price—is due to a joint assumption of price taking and risk aversion. See Sections 2.2 and 3.5 for more details.

[2]Large economies with residual market power are well-founded. Introducing information sales, García and Sangiorgi (2011) show that the monopolistic competition in Kyle (1989) arises naturally. In an economy with endowment shocks, Kyle and Lee (2018) show that if speculation is strong enough relative to hedging, then markets can remain imperfectly competitive as the number of traders grows.

homogeneous economy, I show that equilibrium exists only if an aggregate measure of risk seeking is positive. I call this measure "risk appetite." While it technically reduces to the negative of risk aversion in a homogeneous economy, it is generally a precision-weighted version of the traders' love for risk. It also completely determines several quantities, such as trading intensity, liquidity, price informativeness, and volume.

By specifying a spectrum of diverse risk preferences, I construct a unique equilibrium with trade, setting the stage for the following comparative statics.

There is a pronounced substitution effect between trading intensity and risk seeking. As the risk seekers in the economy increase, all the traders taken together trade less aggressively in response. Given that several economic measures are functions of aggregate trading intensity, this effect has a number of consequences.

Extending the intuition we gain from the toy model, market impact increases in the fraction of risk seekers. Liquidity, therefore, decreases. This property generalizes the result in Kyle (1985) that market impact is inversely related to trading intensity. Kyle's result still holds in a heterogeneous market, even if we account for competition and arbitrary risk preferences. This property is also intuitive—risk seekers trade more aggressively than risk averters, and thus a trader population with more risk seekers has a higher market impact.

As I allude to above, prices are noisier in equilibria with more risk seekers. As risk seeking increases in the aggregate, individuals trade less aggressively in response, and they therefore contribute less information to the order flow. Consequently, the market aggregates the information of all traders to a smaller extent, making the price an overall less accurate signal of dividend information.

What is more, the intuition summarized in Dow and Gorton's terminology of "noise trade" appears explicitly. Using demand diversity as a proxy for trading volume, I show that the agents trade with each other if and only if the price is noisy. Due to the negative effect of risk seeking on trading intensity, it follows that, even though we need some risk seekers to get any trade at all, the more of them we have, the less everyone trades.

## 1.1 Literature review

"One may introduce risk-loving traders."[3] Such is the advice of Tirole (1982), who, together with Milgrom and Stokey (1982), establish early results in the trade literature. The takeaway from these studies is stark. To motivate trade, and therefore also equilibrium, we must appeal to one of the following mechanisms: broadly exogenous supply, such as liquidity shocks and noise trading; more narrowly defined trading needs, such as random endowments and hedging concerns; and relaxed rationality, which may vary from differences of opinion to specific cognitive frictions. The literature develops largely along these three tracks.

A popular way to model noisy markets assumes that the supply of the traded asset varies exogenously. This idea first appears in Grossman (1976), who introduces the concept of stochastic supply as a device that obscures the fundamental information otherwise contained in prices. Grossman and Stiglitz (1980) and Hellwig (1980) develop this idea further, establishing workhorse models in the information economics of financial markets. As Dow and Gorton (2008) point out, there are two economic interpretations of stochastic supply. One interpretation portrays variation in supply as liquidity shocks to investors' personal circumstances, whereas another argues that some investors' trades are driven by irrational, and therefore "noisy," dispositions.

Since Grossman (1976), a number of papers have proposed other theories for price noise. One such approach is that of Diamond and Verrecchia (1981) and Verrecchia (1982a), who introduce random endowments in order to both generate trade and to avoid full revelation

[3]Even though this advice appears explicitly at the bottom of p. 1167 in Tirole (1982), risk seekers are not mentioned again therein.

of the asset's fundamental value. As García and Urošević (2013) discuss, these endowment models rely on noise variance which grows in the number of traders, a point also raised in Kyle and Lee (2018). A related literature—such as Wang (1994), Dow and Gorton (1997), Albuquerque and Miao (2014), and others—replaces endowment shocks with hedging concerns, and discusses a variety of topics that require trade.[4]

Another approach assumes that rationality is bounded, typically due to either imperfect learning, or overconfidence. Constraining investors' learning capacity à la Sims (2003), Peng and Xiong (2006) discuss how inattention affects prices. A different model of imperfect learning is explored in Vives and Yang (2018), where investors process price information subject to receivers' noise as in Myatt and Wallace (2012). Using overconfidence as another type of bounded rationality, Scheinkman and Xiong (2003) discuss how it can support trade with heterogeneous noisy beliefs, while Kyle, Obizhaeva, and Wang (2018) study overconfident individuals who agree to disagree about the precision of each other's information.

There are also papers that relax specific assumptions of rational expectations. One such example is Banerjee and Green (2015), where uninformed investors are uncertain about whether the individuals they trade against are informed traders or noise traders. Other examples are Vives (2011) and Rostek and Weretka (2012); here, traders have correlated asset valuations, and equilibria can be privately revealing.

As Kyle and Lee (2018) discuss, there are several models with residual market impact. One such model is that of section 9 of Kyle (1989), albeit with noise traders and homogeneous risk aversion. A similarity between that model and the one herein is the structure of idiosyncratic noise, although in my model this noise is represented by Brownian increments.[5]

[4]Dow and Gorton (2008) conduct a survey of the literature as of 2008. For more recent reviews, the interested reader can consult García and Urošević (2013) and Han, Tang, and Yang (2016).

[5]The traders are arranged in an increasingly finer grid, and the signal-to-noise ratio of their signals is a function of how fine the grid is. As the economy becomes large this grid converges to a continuum, and the signals converge to a cross-sectional Brownian motion with drift.

Another large economy with market impact is in Rostek and Weretka (2015), but it assumes that agents have different valuations of the traded asset—in my economy the valuations are the same for everyone.

There is an extensive literature on estimating risk attitudes, both empirically and experimentally. A number of studies in that literature find that a minority fraction of people are risk seekers. Coombs and Pruitt (1960) carry out experiments using simple gambles with a population of 99 American subjects—they find that about one third of the participants prefer gambles with higher variance. Ali (1977) estimates the risk preferences of a representative, rational, expected-utility-maximizing agent who has objective knowledge of risky outcomes. Using data from more than $20,000$ horse races in New York, he estimates that the representative race bettor is a risk lover.

In a more recent study, Kachelmeier and Shehata (1992) elicit certainty equivalents for lotteries in experiments under high monetary incentives with a group of 80 Chinese subjects. Their evidence suggests that risk-seeking behavior is present in a fraction of individuals. Holt and Laury (2002) provide measures of relative risk aversion from experiments with 175 American subjects, also with significant monetary incentives. Their estimated risk-aversion coefficients are negative for a small fraction of individuals. Dohmen et al. (2011) use a German survey of $22,000$ respondents combined with a field experiment of 450 subjects. They provide quantitatively similar estimates of fractions of risk-seekers in the German population to what Holt and Laury (2002) find for their American subjects.

Discussing the limit without appealing to Brownian motion, Kyle (1989) points out an intuitive analogy with monopolistic competition à la Dixit and Stiglitz (1977). In monopolistic competition, firms maintain market power by producing slightly differentiated copies of an archetype good. In Kyle (1989) and in my model, we have traders instead of firms, signals instead of differentiated copies, and dividend information instead of an archetype.

## 2 Risk-seeking traders

The economy unfolds in one trading period, comprising $N$ rational informed traders—there are no noise traders, endowment shocks, or hedging concerns. The dividend of the asset is $D \sim \mathcal{N}\left(0, \tau_D^{-1}\right)$, and its price is $P$. Each informed trader $n = 1, \ldots, N$ has mean-variance preferences with risk-preference parameter $\delta$. He observes the signal

$$s_n = D + \varepsilon_n \tag{1}$$

about the dividend, where $\varepsilon_n$, $n = 1, \ldots, N$ is a collection of independent random variables, with distribution $\mathcal{N}\left(0, \tau_n^{-1}\right)$, independently of $D$.

I derive equilibria in two markets: one in which traders observe only signals as in (1), and another, in which they also observe the price. Given the information set $\mathcal{F}_n$, which may be either the $\sigma$-algebra of $s_n$ or that of $(s_n, P)$, trader $n$'s utility is

$$u\left(\pi_n; \mathcal{F}_n\right) = \mathbb{E}\left[\pi_n \middle| \mathcal{F}_n\right] - \frac{1}{2}\delta \mathrm{Var}\left(\pi_n \middle| \mathcal{F}_n\right), \tag{2}$$

where $\pi_n = X_n(D - P)$ is his profit.

In either market, I derive two equilibria: a financial-market equilibrium, in which each trader's precision is exogenous, and an information-acquisition equilibrium, in which trader $i$ faces the precision costs

$$c(\tau_i) = \frac{\tau_i^2}{4\psi}, \tag{3}$$

and which is followed by trading as in the financial-market equilibrium.

## 2.1 A toy model with homogeneous risk seekers

There is a representative market maker who sets the price $P$ equal to his conditional expectation of the dividend given the aggregate order flow. Following standard conjectures, the demand strategy of trader $n$ is linear in his signal,

$$X_n = \beta_n s_n, \tag{4a}$$

and the price is linear in aggregate order flow,

$$P = \lambda \left( \sum_{n=1}^{N} X_n \right). \tag{4b}$$

It is well known that without exogenous noise no equilibrium exists. In particular, for a large class of economies, Milgrom and Stokey (1982) and Tirole (1982) have shown that no trade happens if all traders are rational, as long as these traders are risk neutral or risk averse. Nevertheless, as I show next, this result is overturned if we allow risk seeking. The proofs of the results are in the Appendix.

**Proposition 1** *If $\delta \geq 0$, no equilibrium with trading exists. If $\delta < 0$, a unique symmetric equilibrium with the structure of* (4) *exists, in which*

$$\beta_n = \frac{\tau_n}{-\delta} \tag{5a}$$

*and*

$$\lambda = \frac{-\delta}{\tau_D + \sum_{i=1}^{N} \tau_i}. \tag{5b}$$

*In this equilibrium, the second-order condition of each trader is satisfied. Moreover, the traders' precisions are rationalizable ex-ante. For example, under the precision costs in* (3) *a unique equilibrium with information acquisition exists.*

Given the provisions of the trade literature, it may appear counterintuitive that an equilibrium exists. It is important, however, to point out that risk seeking falls outside the constraints of the traditional no-trade results. In fact, the above theorem suggests that traders who are willing to take on risk are also willing to trade.

Several questions now arise. First, why is there an equilibrium at all? Standard intuition from price-taking models dictates that risk seekers trade very aggressively, and may thus attempt to hold infinite positions in the asset, thereby destroying equilibrium.[6] Second, are risk seekers rational? Can we think of them, that is, as utility-maximizing agents whose second-order conditions not violated?

These questions are connected. Examining the optimal utility the traders,

$$u\left(\pi_n; s_n\right) = \frac{1}{2}\beta_n^2 s_n^2 \left[2\lambda + \delta \mathrm{Var}\left(D - P_{-n} \middle| s_n\right)\right], \tag{6}$$

together with their demand in (5a), we can see that not only they are willing to trade, but their demands are finite and well-defined. Crucially, however, both for rationality and for the willingness to trade, the square bracket in (6) must be positive.

We can recognize the square bracket as the negative of the traders' second-order condition. Similarly to Kyle (1989), it consists of two parts: market impact, and a risk-aversion term; the second-order condition is thus satisfied if the sum of the two parts is overall positive. Negative risk aversion is therefore admissible, as long as market impact can offset it.

To wit, as we can see in (5b), the magnitude of market impact is proportional to the traders' love for risk. This is intuitive, because the more the traders like risk, the more

[6]The first-order condition of a price-taking trader with CARA utility, risk-aversion coefficient $\delta$, information set $\mathcal{F}$, and optimal demand $X$ is $X = \mathbb{E}\left[D - P \middle| \mathcal{F}\right] / \left\{\delta \mathrm{Var}\left(D - P \middle| \mathcal{F}\right)\right\}$. The second-order condition is $-\delta \mathrm{Var}\left(D - P \middle| \mathcal{F}\right)$, which is violated for negative $\delta$. Thus, under price-taking assumptions, negative risk aversion implies that the demand does not correspond to a maximum.

aggressively they trade, amplifying their market impact to the extent that it overwhelms the negative risk aversion term.

As we can further see in (5a), trading intensity is inversely related to the traders' love for risk, reflecting that, being strategic, the traders internalize how they affect prices. Realizing that everyone else also trades aggressively, each individual scales back their trading.[7] Acting as a self-sustaining effect, this strategic response ensures that demands remain finite and that market impact remains positive—positive enough to guarantee rationality. In sum, what is an ex-ante convex utility function becomes concave in equilibrium.

## 2.2 Learning from prices

Here I extend the above results to observable prices. Traders now maximize their utility by observing the price $P$ in addition to their private signal in (1). There is no explicit market maker, and I assume—in contrast to Kyle (1989)—that the market clears deterministically.

More specifically, trader $n$ conditions his demand on $s_n$ and on $P_{-n}$, which stands for the price excluding his impact. As in Kyle (1989), $P_{-n}$ is defined by

$$P = P_{-n} + \lambda_{-n} X_n, \tag{7}$$

with $\lambda_{-n}$ being the slope of trader $n$'s residual supply curve. Following standard methodology, I continue to assume that prices and demands are linear. Letting the demand function of trader $n$ be

$$X_n = \beta_n s_n - \gamma_n P, \tag{8}$$

[7] This happens in a manner similar to Kyle (1985), Subrahmanyam (1991), Holden and Subrahmanyam (1992), Foster and Viswanathan (1996), and others.

the price that clears the market is

$$P = \frac{\sum_{n=1}^{N} \beta_n}{\sum_{n=1}^{N} \gamma_n} \left[ D + \sum_{k=1}^{N} \frac{\beta_k}{\sum_{n=1}^{N} \beta_n} \varepsilon_k \right]. \tag{9}$$

This price function is similar to that in Grossman (1976), but with the difference that traders are neither price takers nor risk averse.

**Proposition 2** *If $\delta \geq 0$, no equilibrium with trading exists. If $\delta < 0$, a unique equilibrium with trading does exist for homogeneous precisions, where the demand coefficients for each trader are*

$$\beta = \frac{\tau N}{-\delta(N-1)} \tag{10a}$$

*and*

$$\gamma = \frac{\tau N + \tau_D}{-\delta(N-1)}, \tag{10b}$$

*and the second-order condition of every trader is satisfied. Moreover, this equilibrium is rationalizable ex-ante; under the homogeneous precision costs in* (3)*, a unique equilibrium with information acquisition exists.*

As above, risk seeking enables an equilibrium with traders who behave as rational optimizers.[8] This equilibrium not only lies outside the scope of Milgrom and Stokey (1982) and Tirole (1982), but it also shows that the intuition in Grossman (1976) is sensitive to assuming that traders take prices as given. If each individual trader recognizes that the price contains his information only if he trades, then he also recognizes that the price summarizes the information of all other traders except him. The price thus becomes a valuable source of information—but only if traders like risk.

[8]The equilibrium with information acquisition is under the assumption that, having chosen a precision, each trader commits to the demand strategy dictated by their chosen precision.

## 2.3 A comparison with noise trading in terms of information costs

Is it possible to distinguish the risk-seeking economy from existing models with trade? One answer to this question is that, as we can see in Proposition 1, the precisions of each trader can be made endogenous. This is in contrast to models with stochastic supply, random endowments, hedging concerns, and heterogeneous priors, as those may present a bottleneck in endogenizing noise due to its inherent assumed exogeneity.[9]

Another answer comes from a direct comparison of the risk-seeking economy with existing models vis-à-vis empirically observable characteristics. While quantities such as signals and trades may be difficult to measure in reality, quantities such as liquidity and information costs do have empirical counterparts.

To simplify exposition, I compare the pair of risk-seeking economies in Sections 2.1 and 2.2 with two alternative one-period economies with noise traders and risk-neutral rational traders. The first alternative economy extends Kyle (1985) to allow for imperfect information in a manner similar to Holden and Subrahmanyam (1992) and Foster and Viswanathan (1996). The second alternative economy is Kyle (1989) with risk neutrality and without uninformed traders.[10] To facilitate discussion, I follow established terminology that refers to models with unobservable prices and perfectly competitive market makers as economies with "market orders," and to models with observable prices and imperfect competition as economies with "limit orders."

**Corollary 3** *Irrespective of whether we consider market orders or limit orders, in the risk-seeking economies liquidity increases when information becomes cheaper. In the noise-trading*

[9] Exceptions include Admati and Pfleiderer (1988) who use discretionary and nondiscretionary liquidity traders, and Han, Tang, and Yang (2016) who reduce price noise to a function of exogenous benefits from trading.

[10] In all economies, the rational traders acquire information ex-ante subject to the same specification of information costs, and the noise in their signals is idiosyncratic. See Sections A.1 and A.2 of the Internet Appendix for detailed derivations of the alternative economies.

*economies, if information is not sufficiently cheap, then liquidity decreases when information becomes cheaper.*

It is important to note that, in all economies, the rational traders acquire more information when information becomes cheaper. Moreover, in all economies, there are two effects of making information cheaper: one direct, and one indirect.

The direct effect makes liquidity increase. When traders acquire more information, the price endogenously trades closer to the fundamental, reducing the market impact of individual traders. The indirect effect, however, makes liquidity decrease. When the traders' information is fixed, each trader internalizes the impact that he and other traders have on the price. As in Holden and Subrahmanyam (1992) and other models, each individual trader must scale back his trading to take everyone's market impact into account. When the traders acquire more information, each trader scales back their trading even more, exactly because everyone's market impact is now bigger.

In the risk-seeking economies the direct effect dominates. When, however, we inject noise traders into an economy, the relationship between liquidity and acquired precision changes drastically. The rational traders internalize their impact on the price, but the noise traders do not. To take this irrationality into account, the rational traders must scale back their trades to a greater extent than in an economy without noise traders. Amplifying the indirect effect, this negative externality that the noise traders impose on the rational traders reduces the incentives to trade aggressively. Liquidity, thus, may decrease when information becomes cheaper. The overall effect is reminiscent of crowding-out effects studied in Goldstein and Yang (2017), Diamond (1985), and Verrecchia (1982b), although in a different context and for different reasons.

Figure 1 shows an illustration of the two pairs of economies. On the left, we have liquidity and trading intensity in the market-order economies, with the risk-seeking economies in solid red, and the noise-trading economies in dashed blue. On the right, we have the same objects for the limit-order economies. The graphs are scaled to make comparing the curves easier.

For all economies, trading intensities increase in $\psi$ because the traders acquire more information as it becomes cheaper. Nonetheless, trading intensities are also concave in $\psi$ because the traders scale back their trading as they acquire more information. This marginal effect is more pronounced with noise traders, making the blue curves more concave than the red.

[Figure 1 here]

## 2.4 What if some traders dislike risk?

One question that the above leaves open is whether it is plausible to assume that every trader likes risk. Indeed, to fully understand whether risk seeking can support trade we must explore what happens when risk seeking coexists with other risk preferences. Nevertheless, modeling this type of heterogeneity results in significant loss of tractability, even if we constrain preferences to be mean-variance. In short, we need a model that is general enough to accommodate heterogeneity, yet simple enough to analyze.[11]

The finance literature at large usually circumvents such issues by using continuum representations.[12] Adopting this approach here, however, comes with its own challenges, which include fully-revealing prices due to perfect competition.[13] What is thus required is a

[11]Section B of the Internet Appendix derives an equilibrium with one risk seeker, $N$ risk-neutral traders, and prices set by market clearing, albeit with some loss of tractability.

[12]See, for example, and Merton (1992) and Dumas and Luciano (2017) for how representing time as a continuum—rather than as a discrete grid—enhances tractability in dynamic settings.

[13]Holden and Subrahmanyam (1992) show that the price becomes fully revealing in a large economy à la Kyle (1985) within the first auction, even when the market contains noise traders. See O'Hara (1995) for further discussion of the dynamic setting, and Lambert, Ostrovsky, and Panov (2018) for recent results on the information-aggregation properties of the one-period setting. García and Urošević (2013) and Kovalenkov and Vives (2014) address this problem by scaling the size of the noise traders.

continuum model with residual market impact—so that we can maintain rationality even with negative risk aversion—and with the additional property that prices are noisy. The monopolistic-competition model in section 9 of Kyle (1989)—which, as García and Sangiorgi (2011) show, arises naturally in financial markets—fits the bill.

I present such an economy next, describing it in continuous notation. Section G of the Internet Appendix derives this economy as a limit of a discrete economy with notation closer to that of Kyle (1989) and García and Sangiorgi (2011).

# 3 An economy with heterogeneous risk preferences

As in Section 2.1, there is one uninformed risk-neutral market maker, but instead of finitely many informed traders there is now a continuum of them. The informed traders have mean-variance preferences which allow for risk seeking, and the structure of information is slightly different, although, as I explain in more detail below, I maintain the usual representation of competitive trading with signals as "truth plus noise." As in Section 2.1, there are no noise traders, endowment shocks, or hedging concerns.

Other than that, the model is a standard economy with one trading period and one asset with dividend $D \sim \mathcal{N}\left(0, \tau_D^{-1}\right)$. Each trader submits their demand of the asset to the market maker, without knowing what demand quantities other agents will submit. The market maker sees only the aggregate order flow, and cannot infer which trader demands which quantity.

Each trader corresponds to a point in the interval $[0, 1)$. Trader $a$ is endowed with a signal $dz_a$ about the liquidating dividend, with

$$dz_a = D\,da + \sqrt{\tau(a)^{-1}}dB_a, \tag{11}$$

where $B$ is a standard Brownian motion in the unit interval independently of $D$, $dB_a$ is the $ath$ Brownian increment, and $\tau$ is a continuous function on the unit interval.[14] I stress that by independence of Brownian increments, the noise $dB_a$ is independent across traders, and that its distribution is $\mathcal{N}(0, da)$. The existing literature treats trader-specific noise as an independent random variable whose distribution is normal, but its variance is a constant that does not scale with respect to $da$. This distinction, although technical, is important from a model-building perspective—in a canonical large economy such independent noises vanish via the Law of Large Numbers. In my setup, however, the trader-specific noises do not vanish; they aggregate in the form of a stochastic integral.

Thinking of $da$ as the size of each infinitesimal trader—so that in the aggregate all traders integrate to one—allows us to write everything in differential form. The demand of trader $a$ is $dX_a$. The market maker sets the price $P$ equal to what he expects the asset value to be given the aggregate order flow, so that

$$P = \mathbb{E}\left[ D \middle| \int_0^1 dX_a \right]. \tag{12}$$

As in Kyle (1985) and Holden and Subrahmanyam (1992), I consider demand strategies linear in signals and prices linear in aggregate order flow. In my setting the demand conjecture is

$$dX_a = \beta(a) dz_a, \tag{13}$$

and the price conjecture is

$$P = \lambda \int_0^1 dX_a. \tag{14}$$

[14] One may recognize the structure in (11) as a Brownian motion with an unknown drift, where instead of the usual "time" argument, the value $a$ takes on a set of values corresponding to the cross section of traders $[0, 1)$. For a similar trick, albeit with a Brownian-bridge process as opposed to the process of (11), and in a different context, the interested reader can consult Gârleanu et al. (2015)

The profit for trader $a$ is $d\pi_a = dX_a(D - P)$. Given the signal $dz_a$, the utility of trader $a$ is

$$du_a = \mathbb{E}\left[d\pi_a \middle| dz_a\right] - \frac{1}{2}\delta(a)\mathrm{Var}\left(d\pi_a \middle| dz_a\right), \tag{15}$$

where $\delta(a)$ measures the risk aversion of trader $a$. I assume that $\delta$ is continuous, and, because we may order traders in the interval without loss of generality, that $\delta$ is an increasing function.

To examine implications for trade, I use the cross-sectional integral of squared realized demands of all traders,

$$\mathcal{V} = \int_0^1 (dX_a)^2, \tag{16}$$

which we can think of as the diversity of demand. In analogy to Admati and Pfleiderer (1988), $\mathcal{V}$ is a quadratic version of the volume of trade that is crossed among all the informed traders.[15] Of course, as Admati and Pfleiderer (1988) point out, measuring trade by looking only at informed demands ignores that the market maker takes the opposite side of any residual demand. A more meaningful measure of trade would therefore incorporate both sides of trade, as in the measure used in Admati and Pfleiderer (1988), Foster and Viswanathan (1993), and others. In my economy the measure that accomplishes this is

$$\mathcal{E} = \max\left\{\left[\int_0^1 \left(dX_a^+\right)^2\right]^{\frac{1}{2}}, \left[\int_0^1 \left(dX_a^-\right)^2\right]^{\frac{1}{2}}\right\}, \tag{17}$$

which we can think of as a continuous Euclidean-norm version of the volume in Admati and Pfleiderer (1988). As I explain in more detail in Corollary 5 below, the notation in (17) corresponds to a measure which is not only well defined, but more importantly, is bounded

[15] On the one hand, no expectation is necessary in (16), because, as I show below, $\mathcal{V}$ is deterministic due to standard properties of quadratic variation of Brownian Motion. On the other hand, we must use squares instead of absolute values, because the absolute variation of Brownian Motion is infinite, which implies that a version of (16) with an absolute value does not exist. I make the analogy between $\mathcal{V}$ and the volume in Admati and Pfleiderer (1988) more precise in a later section of the paper.

both above and below by linear functions of the demand diversity $\mathcal{V}$. We may thus focus on $\mathcal{V}$ as a natural measure of trade in the economy.

## 3.1 Derivation of equilibrium

Because all random variables are Gaussian, the first-order condition guarantees that $dX_a$ is a linear function of $dz_a$. This not only confirms the linear form of the conjecture (13), but also, as a consequence, the linear form of conjecture (14). We may thus follow the usual approach of matching coefficients to obtain the values of $\beta(a)$ and $\lambda$. Doing so, and treating $da$ as an infinitesimally small quantity, yields the following.

**Theorem 4** *Given a precision function $\tau$, if an equilibrium exists, then it must satisfy*

$$\beta(a) = \frac{\tau(a)}{2\rho + \delta(a)} \tag{18a}$$

*and*

$$\frac{1}{\lambda} = \int_0^1 \beta(s)ds + \frac{\tau_D}{\rho}, \tag{18b}$$

*where $\rho$ is the solution to*

$$\rho = \left[\int_0^1 \frac{\tau(s)}{[2\rho + \delta(s)]^2} ds\right]^{-1} \int_0^1 \frac{\tau(s)}{2\rho + \delta(s)} ds, \tag{18c}$$

*under the restriction that $\rho$ and $\int_0^1 \beta(s)ds$ have the same sign. Moreover, the second-order condition of trader $a$ is satisfied if and only if $\delta(a) + 2\rho > 0$.*

A few important comments are in order. First, as we can see in (18a) and (18b), the market-impact parameter $\lambda$ depends on integrals of the trading intensity function $\beta$, which in turn depends on $\rho$. As a result, every equilibrium quantity is a function of $\rho$ alone. Theorem

4 therefore completely characterizes the equilibrium, provided that we can solve (18c), and provided that we have a precision function $\tau$ in hand.

There is a special case in which (18c) simplifies to something we can recognize immediately. If all traders have the same risk aversion coefficient, $\delta(s) = \delta$ for all $s$, then we have

$$\rho = -\delta. \tag{19}$$

Nonetheless, if the traders have different risk aversions, then $\rho$ is affected by their precisions. We can thus think of $\rho$ as a precision-weighted version of how much the traders like risk—I refer to it as "risk appetite."

We may also think of $\rho$ as a version of market impact adjusted for competition and preferences. According to Equation (18b), market impact is one-to-one with risk appetite, modulo a correction in the form of aggregate trading intensity. This correction comes from that there is more than one informed trader, and that the traders are not risk neutral. For example, let us counterfactually assume that we have only one trader, and that he is risk-neutral. We can *informally* argue that, even though we have no noise traders, the model would reduce to a version of Kyle (1985). Trading intensity would then be one half of the liquidity parameter, and market impact would then be proportional to risk appetite.

Second, the aggregate order flow contains a combination of signal noises in the form of $\int_0^1 \beta(a)\sqrt{\tau(a)^{-1}}dB_a$. This stochastic integral acts as an "aggregate price noise", and it is a well-defined random variable, with mean zero and variance

$$\mathcal{A} = \text{Var}\left(\int_0^1 \beta(a)\sqrt{\tau(a)^{-1}}dB_a\right) = \int_0^1 \beta^2(a)\tau(a)^{-1}da. \tag{20}$$

Third, the price inherits the noisy character of aggregate order flow. We can thus think of the signal-to-noise ratio of the price, which I call $\mathcal{Q}$, as price informativeness—this measures how much an uninformed outsider learns about the dividend when he observes the price.

Moreover, as is standard in the literature, we can think of the price as the aggregate signal of all traders. Combining the signal in (11) with the conjectures in (12) and (13) we get

$$P = \int_0^1 \lambda\beta(a)dz_a = \lambda\left(\int_0^1 \beta(a)da\right)D + \lambda\int_0^1 \beta(a)\sqrt{\tau(a)^{-1}}dB_a. \tag{21}$$

Trader $a$'s signal, $dz_a$, carries a coefficient equaling the market-impact parameter times the trader's trading intensity, $\lambda\beta(a)$. Thus, as (18a) shows, signals of traders who have higher precision contribute more to the dividend information conveyed by the price, while signals of traders who are more risk-averse contribute less. These facts conform to intuition that traders with higher precision trade more aggressively, and traders with higher risk aversion trade less aggressively.

Finally, using this framework we can not only think of noise as something that reduces to preferences and precisions, but we can also recognize explicitly that, as Dow and Gorton (1997) argue in their first few paragraphs, noise and trade are two faces of the same coin.[16]

**Corollary 5** *For any equilibrium as described in Theorem 4, the variance of the aggregate price noise is*

$$\mathcal{A} = \frac{\int_0^1 \beta(a)da}{\rho}, \tag{22a}$$

*but it is also equal to demand diversity, i.e.* $\mathcal{A} = \mathcal{V}$. *Volume obeys the following bounds in demand diversity:*

$$\frac{1}{2}\mathcal{V} \leq \mathcal{E}^2 \leq \mathcal{V}. \tag{22b}$$

We can see that demand diversity is a positive constant, and that it equals the variance of the aggregation of the idiosyncratic noises of all traders. This is not surprising—by

[16]As Dow and Gorton (1997) explain in their Section II.B, their model replaces the usual devices of liquidity trading, random endowments, and irrationality by hedging against exogenous income shocks.

definition, demand diversity is the quadratic variation of a Brownian motion in the continuum of traders.[17] Given that aggregate noise $\mathcal{A}$ is identical to demand diversity $\mathcal{V}$, the bounds in (22b) express the following duality: prices are noisy only if rational informed individuals trade, and rational individuals trade only if the price noise provides "enough cover" for their information.

## 3.2 Risk seeking in the heterogeneous economy

I now turn to the question of whether an equilibrium exists—the answer depends on the traders' risk preferences.

**Theorem 6** *For a given precision function $\tau$, if $\delta(s)$ is non-negative for all $s$ then no equilibrium exists in which the second-order condition of all traders is strictly satisfied.*

There are two lessons we can draw from Theorem 6. First, the results of Milgrom and Stokey (1982) and Tirole (1982) extend to the heterogeneous market-order economy. There is no equilibrium if all traders are weakly risk averse, irrespective of how diverse their risk aversions might be. Thus, if an equilibrium exists, then having at least some risk seekers is a necessary condition.

Second, a rational equilibrium must satisfy the traders' second-order condition. As we can see from Theorem 4, this happens if and only if

$$\delta(a) + 2\rho > 0, \tag{23}$$

which shows that negative risk aversion is allowed, as long as it does not exceed $-2\rho$.

[17]This property of demand diversity renders the model falsifiable, assuming that we can gather demand data which conform to the model. According to Corollary 5, two things should happen to the sum of squared trader demands for large numbers of traders: it should be finite and positive, and it should not have any variance.

To summarize, we *might* have an equilibrium if we use risk-seeking preferences. This, of course, leaves open the question of whether an equilibrium *does* exist. As we can see in (18c), however, it is difficult to answer this question in general—the conditions involved do not become tractable until we put some appropriate structure on $\delta$ and $\tau$. I construct such an example next. Some general sufficient conditions on preferences and precisions are in Section E of the Internet Appendix.

### 3.2.1 An example with a linear spectrum of preferences

To obtain a tractable model, I assume that risk preferences have the structure

$$\delta(a) = \chi(a - \phi), \tag{24}$$

where $a$ is in $[0, 1)$, $\chi > 0$, and $\phi < 1$.[18] While I do not restrict its sign, if $\phi$ is positive then it measures how many risk seekers we have in the economy. If $\phi$ is negative, then it is merely a parameter that determines the aggregate risk aversion in the economy. The parameter $\chi$ measures how spread out the risk preferences are, as $\delta(1) - \delta(0) = \chi$. I refer to $\phi$, whenever it is positive, as the "fraction of risk seekers," and to $\chi$ as the "preference spread."

To further simplify the analysis I use homogeneous precisions for all traders, so that $\tau(a) = \tau$ for all $a$.[19]

**Proposition 7** *An equilibrium exists for the market with the risk-preference function of* (24)*, homogeneous precisions, and a positive fraction of risk seekers. In addition, the second-order condition of every trader is satisfied in equilibrium.*

[18] The specification in (24) is a line with intercept $-\phi\chi$ and slope $\chi$; representing a line this way is more convenient for exposition than using intercepts and slopes.

[19] I analyze an equilibrium with endogenous precisions in the following section.

Here we have an example where equilibrium exists with only partial risk seeking. To gain intuition about why, we can reexamine the price function of (21).

We can see that the amount of price noise depends on the trading intensity of different traders, and that, as (18a) shows, trading intensity is inversely related to the coefficient of risk aversion. Therefore, holding the trading intensities of all other traders fixed, the trading intensity of a particular trader is larger when his risk-aversion coefficient is smaller, and all the more so if it is negative. We can thus conclude that equilibrium fails to exist if $\delta \geq 0$ because weak risk aversion does not generate enough price noise. Even partial risk seeking, however, does.

Another way to look at why an equilibrium exists with partial risk seeking is through a graphical interpretation. Integrating (18a), we can see that—holding exogenous quantities fixed—the aggregate trading intensity depends only on $\rho$. The existence of equilibrium thus boils down to whether (18c) has a positive solution in $\rho$. Interpreting (18c) as a fixed-point mapping in $\rho$, an equilibrium exists if its right-hand side crosses the 45° line.

[Figure 2 here]

Figure 2 illustrates this equilibrium. For the preference spectrums of Panel A, Panel B shows the right-hand side of the fixed-point mapping in $\rho$. We can see that $\rho$ cannot be positive without risk seeking, but it can be positive even with small numbers of risk seekers—allowing $\delta(a)$ to become negative for some $a$ shifts (18c) enough to the right to guarantee a crossing.

## 3.3 Information acquisition

I have so far assumed that information is exogenous. Can equilibrium continue to exist if the traders choose how much information they acquire? To address this question, I assume that each trader $a$ faces the cost function

$$c\left(a;\tau(a)\right)=\frac{\tau(a)^2}{4\psi(a)},\tag{25}$$

where $\psi(a)>0$ and continuous on the unit interval, and $\tau(a)$ maximizes

$$\mathbb{E}\left[du_a\right]-c\left(a;\tau(a)\right)da.\tag{26}$$

The first-order condition of each trader yields a relationship between his precision, his idiosyncratic parameters, and the aggregate quantities $\rho$ and $\int_0^1\beta(s)ds$.

**Lemma 8** *In any permissible equilibrium with information acquisition, the endogenous precision of agent a is*

$$\tau(a)=\left[\rho\int_0^1\beta(s)ds+\tau_D\right]^{-1}\frac{\psi(a)}{\delta(a)+2\rho}.\tag{27}$$

Setting the question of existence aside momentarily, we can see that the information acquisition of trader $a$ responds to his idiosyncratic parameters, $\delta(a)$ and $\psi(a)$, in the following manner. A trader with cheaper information costs acquires more information, and a trader who is more risk-averse acquires less information. The risk-aversion effect may seem counterintuitive, because acquiring more information allows each trader to resolve risk. Nevertheless, as is standard in other frameworks with information acquisition, there is also a demand effect; a more risk-averse trader holds less of the asset, which reduces his demand for information. The demand effect overwhelms the risk-resolution effect, and thus a more risk-averse trader overall acquires less information.

Information acquisition responds to both aggregate quantities, $\rho$ and $\int_0^1\beta(s)ds$, in the same manner. Holding risk appetite $\rho$ fixed, each trader acquires less information when aggregate trading intensity $\int_0^1\beta(s)ds$ increases (mutatis mutandis for when risk appetite increases.) We thus have a certain type of substitution in information acquisition—when other traders trade more aggressively as a whole or when they altogether have a higher impact on the price, each trader acquires less information in response.

Combining the endogenous precision function in (27) with each trader's trading intensity in (18a) makes it possible to close the model by deriving the integrals of precision that appear in the equilibrium condition for $\rho$. The resulting equilibrium has the following structure.

**Theorem 9** *If an equilibrium with information acquisition exists, then it must satisfy*

$$\rho = \left[\int_0^1 \frac{\psi(s)}{[\delta(s)+2\rho]^3} ds\right]^{-1} \int_0^1 \frac{\psi(s)}{[\delta(s)+2\rho]^2} ds \tag{28a}$$

*and*

$$\int_0^1 \beta(s)ds = \left[\rho \int_0^1 \beta(s)ds + \tau_D\right]^{-1} \int_0^1 \frac{\psi(s)}{[\delta(s)+2\rho]^2} ds, \tag{28b}$$

*under the restriction that $\rho$ and $\int_0^1 \beta(s)ds$ have the same sign.*

Conditions (28a) and (28b) of Theorem 9 are a system of two equations in two unknowns, $\rho$ and $\int_0^1 \beta(s)ds$. These equations are quite general, and they depend only on exogenous parameters: the precision of the dividend $\tau_D$, the preference-coefficient function $\delta$, and the inverse marginal-cost function $\psi$.

Returning to the question of whether equilibrium continues to exist, I first show that risk seeking is again necessary.

**Theorem 10** *For a given inverse marginal-cost function $\psi$, if $\delta(a)$ is non-negative for all $a$ then no equilibrium with information acquisition exists.*

As we can see, the classic no-trade results continue to hold with information acquisition. In fact, with information acquisition the conclusion of Theorem 6 becomes stronger, as Theorem

10 excludes the possibility of equilibrium even if we allowed some traders to violate their second-order conditions, or if we used heterogeneous information costs. Nevertheless, as I show with an example, we can have equilibrium with risk seeking.

### 3.3.1 Information acquisition with a linear preference spectrum

For linear risk preferences and homogeneous costs ($\psi(a) = \psi > 0$ for all $a$) we have the following result.

**Proposition 11** *With the risk-preference function of* (24)*, homogeneous information costs, and a positive fraction of risk seekers, a unique equilibrium with information acquisition exists, in which the second-order condition of every trader is satisfied.*

*Moreover, in this equilibrium liquidity always increases when information becomes cheaper. In contrast, in a version of the economy where instead of risk seekers we have noise traders and weakly risk averse rational traders, if information is not sufficiently cheap, then liquidity decreases when information becomes cheaper.*

As we can see, not only does the result in Proposition 7 continue to hold with endogenous precisions, but it also strengthens to guarantee a unique equilibrium.[20] I show this equilibrium in Panel C of Figure 2 for the same parameters as for Panel B, but with endogenous precisions. Similarly to the equilibrium with homogeneous exogenous precisions, making risk aversion negative for some traders shifts the curve on the right-hand side of (28a) enough to the right to guarantee a crossing with the 45° line.

Juxtaposing Proposition 11 with Corollary 3, if we allow some traders in the risk-seeking economy to dislike risk, then we come to the same conclusion: as information becomes

[20]Under the diverse risk preferences of (24), it becomes possible to write down the equilibrium as polynomial equations, and to appeal to well-known results from analysis to guarantee uniqueness—see the proof for more details. For similar reasons, it is hard to guarantee uniqueness without endogenous precisions because the equilibrium reduces to a transcendental equation.

cheaper, liquidity always increases. If, however, we replace the risk seekers with noise traders, then the remaining rational traders have to take the noise traders' exogenous behavior into account. Crowding out the rational traders' incentives to trade aggressively, this externality can make liquidity decrease as information becomes cheaper.

### 3.4 Comparative statics with respect to risk seeking

How does risk seeking affects the properties of equilibrium beyond merely enabling it? To address this point, I explore some comparative statics with respect to the risk-preference parameters.

**Corollary 12** *In the equilibrium with information acquisition,*

*(i) risk appetite increases in $\phi$ and in $\chi$,*

*(ii) the aggregate trading intensity $\int \beta$ decreases in $\phi$ and in $\chi$,*

*(iii) the trading intensity of every trader decreases in $\phi$ and in $\chi$, and*

*(iv) liquidity, demand diversity, and price informativeness decrease in $\phi$ and in $\chi$.*

The economy changes in the same manner as we increase either the fraction of risk seekers $\phi$ or the preference spread $\chi$. In this sense, changing either $\phi$ or $\chi$ yields qualitatively equivalent comparative statics. This happens because, in intuitive terms, both $\phi$ and $\chi$ measure the general love for risk in the trader population—$\phi$ measures how many risk seekers we have, and $\chi$ measures how much more risk tolerant the risk seekers are in relation to the risk averters.

That risk appetite increases in the risk-seeking characteristics $\phi$ and $\chi$ is therefore a natural consequence of increasing the traders' love for risk. For example, as Figure 2 confirms, when $\phi$ increases, the curve of the fixed-point mapping shifts to the right; given that the 45° line remains fixed, this shift implies that risk appetite increases in equilibrium.

Aggregate trading intensity decreases in the risk-seeking characteristics due to a substitution effect. If we increase the risk seeking of some traders then we increase the market impact of every trader, because market impact is an aggregate quantity. Being forced to accommodate the increased aggression of others, the traders whose risk seeking does not change then respond by lowering their trading intensity so as to reveal less of their private information.

In fact, this substitution effect is so strong that it holds for every trader in the economy, regardless of whether they are risk-seeking or risk-averse. At least in principle, the aggregate trading intensity could decrease even if the trading intensity of some traders increased. But as Corollary 12 shows, the trading intensity of *every* trader decreases in the risk-seeking characteristics. This happens because the trading intensity of a specific trader is mostly driven by adverse risk-appetite effects.

More specifically, the trading intensity of trader $a$ is

$$\beta(a) = \frac{\psi(a)}{[\delta(a) + 2\rho]^2 \left[\rho \int_0^1 \beta(s)ds + \tau_D\right]}. \tag{29}$$

As we can see in (29), any given individual trades more aggressively if all others taken together trade less aggressively. Holding the equilibrium $\rho$ constant, we thus expect that increasing the risk seeking of enough traders would increase an individual's trading intensity through its negative effect on aggregate trading intensity. Nonetheless, an individual's trading intensity is inversely related to risk appetite in the aggregate. In addition, the magnitudes coming from increased risk appetite dominate all other effects, making the trading intensity of individuals an overall decreasing function of risk-seeking characteristics.[21]

Reflecting the above intuition that market impact increases because traders behave more aggressively, liquidity decreases in the risk-seeking characteristics. Moreover, as we can see in Corollary 5, demand diversity is directly related to aggregate trading intensity, and inversely related to risk appetite. That it decreases in $\phi$ and $\chi$ is therefore an immediate consequence of first two parts of Corollary 12.

Finally, a straightforward calculation—see (C.20) for details—shows that price informativeness increases in both risk appetite and aggregate trading intensity. This is intuitive; prices aggregate information more accurately either whenever traders are more aggressive, or whenever demands influence prices to a greater extent. Herein we have two opposing effects: risk appetite increases in the risk-seeking characteristics, but trading intensity decreases in them. Juxtaposing these two forces with intuition from noise-trading models with risk aversion (Subrahmanyam, 1991, p. 427)—and thinking of risk seeking as the pure opposite of risk aversion—we may expect that price informativeness would increase in risk seeking. Nevertheless, price informativeness decreases in the risk-seeking characteristics. The substitution effect that risk seeking induces in trading intensity is so strong that it overwhelms any direct effects in risk appetite.

## 3.5 Extension to learning from prices

Trader $a$ is endowed with the pair of signals

$$dz_a = D\ da + \sqrt{\tau(a)^{-1}} dB_a^z \tag{30a}$$

$$d\zeta_a = P\ da - \sqrt{\pi(a)^{-1}} dB_a^\zeta, \tag{30b}$$

[21]The denominator of $\beta(a)$ in (29) is a function of the third power of adjusted market impact $\rho$, but it is only a linear function of aggregate trading intensity and risk preferences.

where $B^z$ and $B^\zeta$ are two independent Brownian motions on $[0, 1)$.[22] With demand strategies linear in signals,

$$dX_a = \beta(a)dz_a - \gamma(a)d\zeta_a, \tag{31}$$

and deterministic market clearing

$$\int_0^1 dX_s = 0, \tag{32}$$

we obtain

$$P = \lambda \left[ \left( \int_0^1 \beta(s)ds \right) D + \int_0^1 \beta(s)\sqrt{\tau(s)^{-1}}dB_s^z + \int_0^1 \gamma(s)\sqrt{\pi(s)^{-1}}dB_s^\zeta \right], \tag{33}$$

where $\lambda$ is the inverse of the aggregate slope of demand with respect to price.

**Theorem 13** *Given precision functions $\tau$ and $\pi$, if the ratio $\frac{\pi(s)}{\tau(s)}$ does not depend on $s$, then an equilibrium exists if and only if one exists without price observations as in Theorem 4. In that case,*

$$\beta(a) = \frac{1}{\mu}\beta_*(a), \tag{34a}$$

$$\gamma(a) = \left(1 - \frac{1}{\mu}\right)\omega\beta_*(a), \tag{34b}$$

*and*

$$\frac{1}{\lambda} = \frac{1}{\mu\lambda_*}, \tag{34c}$$

*where $\omega = \frac{\pi(s)}{\tau(s)}$ is a constant,*

$$\mu = 1 + \omega^{-1}\left(1 + \frac{\tau_D}{\rho_* \int_0^1 \beta_*(a)da}\right), \tag{34d}$$

[22] The minus sign in the volatility of (30b) is intended to simplify notation, and is without loss of generality because $dB_a^\zeta$ and $-dB_a^\zeta$ are equal in distribution.

*and the starred variables are the equilibrium quantities in the market without price observations.*

*If the ratio $\omega(s)$ depends on $s$, then no equilibrium in which the second-order condition of every trader is satisfied exists unless one exists without price observations. In that case, the equilibrium with price observations is the solution to a system of two equations shown in more detail in Theorem 14 of the Internet Appendix.*

*In the limit of fully observable prices ($\pi \to \infty$) $\mu$ converges to one, $\rho$ converges to the solution of* (18c)*, and $\beta$ and $\gamma$ converge to finite and non-zero functions.*

The main takeaway from Theorem 13 is that a rational equilibrium with any amount of learning from prices exists only if one does without any learning from prices. And because, as I argue above, risk seeking is necessary for an equilibrium with market orders, it follows that risk seeking is also necessary for an equilibrium with limit orders.

Moreover, under certain conditions, the equilibrium with price observations is a scaled version of the equilibrium without price observations. Similarly to the "systematic attention" model of Vives and Yang (2018), we can think of the precision $\pi(s)$ as measuring the attention that traders allocate to prices, but with an idiosyncratic interpretation. In this respect, price noise resembles what Black (1986) calls "noise in the sense of a large number of small events." What is more, this type of aggregation permits a relaxed version of the strong-form efficiency in Fama (1970)—prices aggregate all available information, but they do not fully reveal dividend information.

Interpreting $\omega$ as a proxy of the proportion of attention that traders allocate to prices, and assuming that every trader pays the same proportion of attention to prices, we can think of $\omega$ as a parameter that influences the properties of the market independently of the underlying unscaled equilibrium. In particular, because $\mu$ is inversely related to $\omega$, it follows that trading intensity increases as traders pay more attention to prices. Liquidity thereby increases, reflecting that traders have more information at their disposal.

As attention to prices keeps increasing, we recover a model with limit orders in the spirit of Kyle (1989). As we can see in Figure 3, as $\omega$ keeps increasing we converge to a well-defined limit, and the model becomes a version of the economy in Section 3, but one in which traders condition on the price explicitly. We may thus interpret the market with finite $\omega$ as a relaxed version of the market with fully observable prices, with the lesson that prices become more liquid when people pay more attention to them.

# 4 Conclusion

I present a rational explanation of trade and noise in financial markets. To have an inefficient market, no noise is necessary other than in the traders' signals. What is necessary, however, is that some traders like risk, not only enabling trade, but also providing enough noise for everyone to hide their information. Such risk seekers are rational because, even though their preferences are convex ex-ante, their market impact ensures that their preferences are concave in equilibrium.

To convey the main intuition I use an economy with homogeneous risk seekers. To explore how risk seekers affect the economy tractably, I develop a version of the model that allows for heterogeneous risk preferences. Using an example, I show that, even though we need risk seekers to get any trade at all, they affect several economic measures non-trivially: liquidity, trading intensity, and price informativeness decrease, both in how many risk seekers there are, and in how much they like risk.

As my economy has one trading period and one asset, extending it intertemporally and to multiple assets may yield new insights. Moreover, I follow Kyle (1985) and Kyle (1989) in assuming that all traders value assets in the same way. It may be interesting to study risk seeking with other valuation structures, such as with the correlated values of Vives (2011), and with the equi-commonal values of Rostek and Weretka (2012). It may also be interesting to use correlated errors in traders' signals, similarly to Foster and Viswanathan (1996) and Manzano and Vives (2011). Finally, in the heterogeneous economy I model the traders' signals as a cross-sectional Itô process of the simplest form—it may thus be fruitful to pursue more general versions of the traders' signals. I leave such questions for future work.

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# I Homogeneous risk preferences

**Proof of Proposition 1.**

The first-order condition implies

$$X_n = \frac{\mathbb{E}\left[D - P_{-n} \middle| s_n\right]}{2\lambda + \delta \text{Var}\left(D - P_{-n} \middle| s_n\right)}, \tag{35}$$

where $P_{-n} = \lambda \sum_{\substack{i=1 \\ i \neq n}}^{N} X_i$ is the price excluding the demand of trader $n$. From (4b) we obtain

$$\lambda = \frac{\tau_D^{-1} \sum_{i=1}^{N} \beta_i}{\tau_D^{-1} \left(\sum_{i=1}^{N} \beta_i\right)^2 + \sum_{i=1}^{N} \beta_i^2 \tau_i^{-1}}, \tag{36}$$

while by writing out the conditional moments in (35) and matching coefficients with (4a) we obtain

$$\beta_n = \frac{\tau_n \left(1 - \lambda \sum_{\substack{i=1 \\ i\neq n}}^N \beta_i\right)}{2\lambda(\tau_D + \tau_n) + \delta \left[\left(1 - \lambda \sum_{\substack{i=1 \\ i\neq n}}^N \beta_i\right)^2 + (\tau_D + \tau_n)\lambda^2 \sum_{\substack{i=1 \\ i\neq n}}^N \beta_i^2 \tau_i^{-1}\right]}. \tag{37}$$

I conjecture that (5a) holds; by using it in (36) we get

$$\lambda = \frac{-\delta}{\tau_D + \sum_{i=1}^N \tau_i}, \tag{38}$$

which proves that (5b) holds as long as (5a) holds. Rearranging (37) we have

$$\beta_n = \frac{\tau_n \left(1 - \lambda \sum_{i=1}^N \beta_i\right)}{2\lambda\tau_D + \lambda\tau_n + \delta \left[\left(1 - \lambda \sum_{\substack{i=1 \\ i\neq n}}^N \beta_i\right)^2 + (\tau_D + \tau_n)\lambda^2 \sum_{\substack{i=1 \\ i\neq n}}^N \beta_i^2 \tau_i^{-1}\right]}. \tag{39}$$

By (5a) and (38), we have the auxiliary relations

$$1 - \lambda \sum_{i=1}^N \beta_i = \frac{\tau_D}{\tau_D + \sum_{i=1}^N \tau_i}, \tag{40a}$$

and

$$\lambda^2 \sum_{i=1}^N \beta_i^2 \tau_i^{-1} = \frac{\sum_{i=1}^N \tau_i}{\left(\tau_D + \sum_{i=1}^N \tau_i\right)^2}. \tag{40b}$$

The denominator of (39) is

$$\lambda\left(2\tau_D + \tau_n\right) + \delta \left(1 - \lambda \sum_{i=1}^N \beta_i + \lambda\beta_n\right)^2 + \delta(\tau_D + \tau_n)\lambda^2 \left(\sum_{i=1}^N \beta_i^2 \tau_i^{-1} - \beta_n^2 \tau_n^{-1}\right) =$$

$$
\begin{aligned}
&\lambda\left(2\tau_D+\tau_n\right)+\delta\left(1-\lambda\sum_{i=1}^{N}\beta_i\right)^2+2\delta\left(1-\lambda\sum_{i=1}^{N}\beta_i\right)\lambda\beta_n+\delta\lambda^2\beta_n^2\\
&\quad+\delta(\tau_D+\tau_n)\lambda^2\sum_{i=1}^{N}\beta_i^2\tau_i^{-1}-\delta(\tau_D+\tau_n)\lambda^2\beta_n^2\tau_n^{-1}=\\
&\frac{\delta}{\left(\tau_D+\sum_{i=1}^{N}\tau_i\right)^2}\left[-\left(2\tau_D+\tau_n\right)\left(\tau_D+\sum_{i=1}^{N}\tau_i\right)+\tau_D^2+2\tau_D\tau_n+\tau_n^2\right.\\
&\quad\left.+\left(\tau_D+\tau_n\right)\sum_{i=1}^{N}\tau_i-\left(\tau_D+\tau_n\right)\tau_n\right]=\delta\frac{-\tau_D}{\tau_D+\sum_{i=1}^{N}\tau_i}. \qquad (41)
\end{aligned}
$$

where the first equality follows by expanding the square, and the second equality follows by the relations in (40). Using (41) and (40a) in (39) we have

$$
\beta_n=\frac{\tau_n}{-\delta}, \qquad (42)
$$

which verifies the conjecture in (5a). The second-order condition is

$$
2\lambda_{-n}+\delta\mathrm{Var}\left(D-P_{-n}\middle|s_n\right)>0, \qquad (43)
$$

which holds because, by (5a) and (5b),

$$
\begin{aligned}
2\lambda+\delta\mathrm{Var}\left(D-P_{-n}\middle|s_n\right)&=2\lambda+\delta\left(1-\lambda\sum_{\substack{i=1\\i\neq n}}^{N}\beta_i\right)^2\mathrm{Var}\left(D\middle|s_n\right)+\delta\lambda^2\sum_{\substack{i=1\\i\neq n}}^{N}\frac{\beta_i^2}{\tau_i}\\
&=\delta\left[\frac{-2}{\tau_D+\sum_{i=1}^{N}\tau_i}+\left(\frac{\tau_D+\tau_n}{\tau_D+\sum_{i=1}^{N}\tau_i}\right)^2\frac{1}{\tau_n+\tau_D}+\frac{\sum_{\substack{i=1\\i\neq n}}^{N}\tau_i}{\left(\tau_D+\sum_{i=1}^{N}\tau_i\right)^2}\right]\\
&=\frac{-\delta}{\tau_D+\sum_{i=1}^{N}\tau_i}>0. \qquad (44)
\end{aligned}
$$

The information-acquisition problem of trader $n$ is

$$\max_{\tau_n} \mathbb{E}\left[u\left(\pi_n; s_n\right)\right] - c(\tau_n). \tag{45}$$

Equations (2) and (35) together imply

$$\mathbb{E}\left[u\left(\pi_n; s_n\right)\right] = \frac{1}{2}\mathbb{E}\left[X_n^2\right]\left[2\lambda + \delta \text{Var}\left(D - P_{-n}\middle| s_n\right)\right]. \tag{46}$$

Using the optimal demand function, we can write the expected utility of trader $n$ as

$$\mathbb{E}\left[u\left(\pi_n; s_n\right)\right] = \frac{1}{2}\frac{\tau_n^2}{\delta^2}\left(\frac{1}{\tau_D} + \frac{1}{\tau_n}\right)\left[2\lambda + \delta \text{Var}\left(D - P_{-n}\middle| s_n\right)\right] = \frac{1}{-2\delta\tau_D}\frac{\tau_n\left(\tau_n + \tau_D\right)}{\tau_D + \sum_{i=1}^{N}\tau_i} \tag{47}$$

where the second equality is due to (44). If $\delta > 0$, then the traders are strictly better off without trading; if $\delta = 0$ no equilibrium exists because the traders' utility and their trading intensity diverges.

Taking the first order condition of trader $n$'s information acquisition problem while holding every other trader's choice fixed we obtain

$$\frac{1}{-\delta\tau_D}\frac{\left(\tau_D + \tau_n\right)^2 + \left(\tau_D + 2\tau_n\right)\sum_{\substack{i=1\\ i\neq n}}^{N}\tau_i}{\left(\tau_D + \tau_n + \sum_{\substack{i=1\\ i\neq n}}^{N}\tau_i\right)^2} = \frac{\tau_n}{\psi}, \tag{48}$$

and setting $\tau_i = \tau$ for all $i$ we get

$$\frac{\delta\tau_D}{\psi}N^2\tau^3 + \left(2\frac{\delta\tau_D^2}{\psi}N + 2N - 1\right)\tau^2 + \left(\frac{\delta\tau_D^2}{\psi} + N + 1\right)\tau_D\tau + \tau_D^2 = 0. \tag{49}$$

Equation (49) has a unique solution by Descartes' rule of signs. In particular, because $\delta < 0$ and $\tau_D > 0$, the only possibility for a multiple positive root requires that the coefficient of

$\tau^2$ is positive and that the coefficient of $\tau$ is negative, but it is straightforward to show that this contradicts that $N$ is positive. ■

**Proof of Proposition 2.** The profit of trader $n$ is $\pi_n = X_n(D - P)$, and his utility is

$$u(\pi_n; s_n, P_{-n}) = X_n \mathbb{E}\left[D - P_{-n} \middle| s_n, P_{-n}\right] - \lambda_{-n} X_n^2 - \frac{1}{2}\delta X_n^2 \mathrm{Var}\left(D \middle| s_n, P_{-n}\right). \tag{50}$$

The market clears deterministically,

$$\sum_{n=1}^{N} X_n = 0, \tag{51}$$

from which we obtain

$$P = \lambda\left(D \sum_{n=1}^{N} \beta_n + \sum_{k=1}^{N} \beta_k \varepsilon_k\right) \tag{52}$$

where

$$\lambda = \frac{1}{\sum_{n=1}^{N} \gamma_n}. \tag{53}$$

Following Kyle (1989), it is straightforward to show that

$$X_n = \frac{\mathbb{E}\left[D \middle| s_n, P_{-n}\right] - P_{-n}}{2\lambda_{-n} + \delta \mathrm{Var}\left(D \middle| s_n, P_{-n}\right)}, \tag{54}$$

where $\lambda_{-n}$ is

$$\lambda_{-n} = \frac{1}{\sum_{\substack{k=1 \\ k \neq n}}^{N} \gamma_k}, \tag{55}$$

and $P_{-n}$ can be written as

$$P_{-n} = \lambda_{-n}\left[D \sum_{\substack{k=1 \\ k \neq n}}^{N} \beta_k + \sum_{\substack{k=1 \\ k \neq n}}^{N} \beta_k \varepsilon_k\right]. \tag{56}$$

It is important to calculate expectations for each trader excluding their impact on the price (we otherwise get back to the conundrums in Grossman (1976), which rests on traders being price takers.) By the projection theorem we have

$$\mathbb{E}\left[D\middle|s_n, P_{-n}\right] = b_n s_n + c_n P_{-n} \tag{57}$$

where

$$b_n = \frac{\tau_n}{\tau_D + \tau_n + \frac{\left(\sum_{\substack{k=1\\k\neq n}}^N \beta_k\right)^2}{\sum_{\substack{k=1\\k\neq n}}^N \frac{\beta_k^2}{\tau_k}}}, \tag{58a}$$

$$c_n = \sum_{\substack{k=1\\k\neq n}}^N \gamma_k \frac{\frac{\sum_{\substack{k=1\\k\neq n}}^N \beta_k}{\sum_{\substack{k=1\\k\neq n}}^N \frac{\beta_k^2}{\tau_k}}}{\tau_D + \tau_n + \frac{\left(\sum_{\substack{k=1\\k\neq n}}^N \beta_k\right)^2}{\sum_{\substack{k=1\\k\neq n}}^N \frac{\beta_k^2}{\tau_k}}}, \tag{58b}$$

and

$$\mathrm{Var}\left(D\middle|s_n, P_{-n}\right) = \frac{1}{\tau_D + \tau_n + \frac{\left(\sum_{\substack{k=1\\k\neq n}}^N \beta_k\right)^2}{\sum_{\substack{k=1\\k\neq n}}^N \frac{\beta_k^2}{\tau_k}}}. \tag{58c}$$

Combining (54), (7), and (57) we get

$$X_n = \frac{b_n s_n + (c_n - 1)\left(P - \lambda_{-n} X_n\right)}{2\lambda_{-n} + \delta \mathrm{Var}\left(D\middle|s_n, P_{-n}\right)} \Rightarrow X_n = \frac{b_n s_n - (1 - c_n)\, P}{\lambda_{-n}\left(1 + c_n\right) + \delta \mathrm{Var}\left(D\middle|s_n, P_{-n}\right)}. \tag{59}$$

Matching coefficients in (8) and (59) we obtain

$$\beta_n = \frac{b_n}{\lambda_{-n}\left(1+c_n\right)+\delta\mathrm{Var}\left(D\middle|s_n,P_{-n}\right)} \tag{60a}$$

and

$$\gamma_n = \frac{\left(1-c_n\right)}{\lambda_{-n}\left(1+c_n\right)+\delta\mathrm{Var}\left(D\middle|s_n,P_{-n}\right)}. \tag{60b}$$

Under homogeneous precisions ($\tau_n = \tau$ for all $n$), using (58) and (55) in (60a) yields

$$\beta = \frac{\tau}{N\tau+\tau_D}\frac{1}{\frac{1}{(N-1)\gamma}\left[1+\frac{(N-1)\tau}{\frac{\beta}{\gamma}(N\tau+\tau_D)}\right]+\delta\frac{1}{N\tau+\tau_D}} \quad\Rightarrow\quad \gamma = \frac{\tau N+\tau_D}{-\delta(N-1)}, \tag{61}$$

which establishes (10b), while using (58) and (55) in (60b) yields

$$\gamma = \frac{1-\frac{(N-1)\tau}{\frac{\beta}{\gamma}(N\tau+\tau_D)}}{\frac{1}{(N-1)\gamma}\left[1+\frac{(N-1)\tau}{\frac{\beta}{\gamma}(N\tau+\tau_D)}\right]+\delta\frac{1}{N\tau+\tau_D}} \quad\Rightarrow\quad \frac{\beta}{\gamma} = \frac{\tau N}{\tau N+\tau_D}, \tag{62}$$

which, after using (61), establishes (10a). The coefficients $\beta$ and $\gamma$ are both positive if and only if $\delta < 0$; moreover, the (negative of the) traders' second-order condition is

$$2\lambda_{-n}+\delta\mathrm{Var}\left(D\middle|s_n,P_{-n}\right) = \frac{2}{(N-1)\gamma}+\delta\frac{1}{N\tau+\tau_D} = -\delta\frac{1}{N\tau+\tau_D} > 0, \tag{63}$$

which holds if and only if $\delta < 0$. Moreover, the ex-ante utility of each trader is

$$\begin{aligned}\mathbb{E}\left[u\left(\pi_n;s_n,P_{-n}\right)\right] &= \mathbb{E}\left[X_n\mathbb{E}\left[D-P_{-n}\middle|s_n,P_{-n}\right]-\lambda_{-n}X_n^2-\frac{1}{2}\delta X_n^2\mathrm{Var}\left(D\middle|s_n,P_{-n}\right)\right]\\ &= \frac{1}{2}\mathbb{E}\left[X_n^2\right]\left[2\lambda_{-n}+\delta\mathrm{Var}\left(D\middle|s_n,P_{-n}\right)\right] = \frac{1}{-\delta(N-1)}\frac{\tau N}{\tau N+\tau_D},\end{aligned} \tag{64}$$

by the law of iterated expectations, the first-order condition, Equation (63), and because

$$\mathbb{E}\left[X_n^2\right] = \mathbb{E}\left[\beta^2 s_n^2 - 2\beta\gamma s_n P + \gamma^2 P^2\right] = \beta^2 \frac{N-1}{\tau N} = \frac{\tau N}{\delta^2 (N-1)}. \tag{65}$$

Equation (64) shows that the agents trade profitably only if $\delta < 0$; with $\delta = 0$ the equilibrium does not exist because $\beta$ and $\gamma$ diverge, and if $\delta > 0$ everyone is strictly better off not trading. The case of heterogeneous precisions with $\delta \geq 0$ is covered by Tirole (1982).

To justify the equilibrium with homogeneous precisions, suppose that each trader $n$ faces the information cost function in (3). His ex-ante utility is

$$\begin{aligned}
&\mathbb{E}\left[u\left(\pi_n; s_n, P_{-n}\right)\right] = \frac{1}{2}\mathbb{E}\left[X_n^2\right]\left[2\lambda_{-n} + \delta \mathrm{Var}\left(D \middle| s_n, P_{-n}\right)\right] \\
&= \frac{1}{2}\left[\beta_n^2\left(\frac{1}{\tau_n} + \frac{1}{\tau_D}\right) - 2\beta_n\gamma_n \frac{\frac{\beta_n + \sum_{\substack{k=1\\k\neq n}}^N \beta_k}{\tau_D} + \frac{\beta_n}{\tau_n}}{\left(\sum_{\substack{k=1\\k\neq n}}^N \gamma_k + \gamma_n\right)} + \gamma_n^2 \frac{\frac{\left(\beta_n + \sum_{\substack{k=1\\k\neq n}}^N \beta_k\right)^2}{\tau_D} + \frac{\beta_n^2}{\tau_n} + \sum_{\substack{k=1\\k\neq n}}^N \frac{\beta_k^2}{\tau_k}}{\left(\sum_{\substack{k=1\\k\neq n}}^N \gamma_k + \gamma_n\right)^2}\right] \\
&\qquad \times \left[\frac{2}{\sum_{\substack{k=1\\k\neq n}}^N \gamma_k} + \frac{\delta}{\tau_D + \tau_n + \frac{\left(\sum_{\substack{k=1\\k\neq n}}^N \beta_k\right)^2}{\sum_{\substack{k=1\\k\neq n}}^N \frac{\beta_k^2}{\tau_k}}}\right]
\end{aligned} \tag{66}$$

Because each trader $k$ commits to $\beta_k$ and $\gamma_k$ after choosing $\tau_k$, it follows by inspection of (58) and (60), that if we fix $(\beta_k, \gamma_k, \tau_k)$ for $k \neq n$, changes in the utility in (66) happen only through $\tau_n$ (with $\beta_n$ and $\gamma_n$ as functions of $\tau_n$.) We may thus take the first-order condition of (66) holding $(\beta_k, \gamma_k, \tau_k)$ for $k \neq n$ as constants. Doing so we get

$$\frac{\tau_n}{2\psi} = \frac{d}{d\tau_n}\mathbb{E}\left[u\left(\pi_n; s_n, P_{-n}\right)\right] = \frac{1}{2\tau_D}\left(\frac{\sum_{\substack{k=1\\k\neq n}}^{N}\frac{\beta_k^2}{\tau_k}}{2\left(\sum_{\substack{k=1\\k\neq n}}^{N}\beta_k\right)^2 + \sum_{\substack{k=1\\k\neq n}}^{N}\frac{\beta_k^2}{\tau_k}\left[\delta\sum_{\substack{k=1\\k\neq n}}^{N}\gamma_k + 2\left(\tau_n+\tau_D\right)\right]}\right)^2$$
$$\times\left\{\delta\left[\left(\sum_{\substack{k=1\\k\neq n}}^{N}\beta_k - \sum_{\substack{k=1\\k\neq n}}^{N}\gamma_k\right)^2 + \tau_D\sum_{\substack{k=1\\k\neq n}}^{N}\frac{\beta_k^2}{\tau_k}\right] + 2\tau_D\sum_{\substack{k=1\\k\neq n}}^{N}\gamma_k\right\} \quad (67)$$

and setting $\tau_k = \tau$ we get

$$\frac{N(N-2)\tau + (N-1)\tau_D}{-\delta(N-1)\left(N\tau+\tau_D\right)^2} = \frac{\tau}{\psi}. \quad (68)$$

Rearranging (68) we have

$$\delta N^2(N-1)\tau^3 + 2\delta N(N-1)\tau_D\tau^2 + \left[\delta(N-1)\tau_D^2 + \psi N(N-2)\right]\tau + \psi(N-1)\tau_D = 0. \quad (69)$$

Because $\delta < 0$, the coefficients of the polynomial in (69) switch signs only once, either between the quadratic and the linear term, or between the linear and constant terms. In either case, by Descartes's rule of signs, (69) has a unique positive solution for $\tau$. ■

**Proof of Corollary 3.** Here I prove the comparative statics for the risk-seeking economies. Because the economies with noise traders are standard, I relegate the proofs for those to Section A of the Internet Appendix.

**Unobservable prices and competitive market makers** Let $F(\tau,\psi)$ denote the left-hand side of (49). $F(\tau,\psi)$ decreases in $\tau$ in equilibrium because $\tau$ is the unique positive solution of a cubic polynomial with a negative leading coefficient ($\delta < 0$.) By the implicit function theorem we obtain

$$\frac{d\tau}{d\psi} = -\frac{\frac{\partial}{\partial\psi}F}{\frac{\partial}{\partial\tau}F} = \frac{\delta\tau_D\tau\left(\tau_D+N\tau\right)^2}{\psi^2\frac{\partial}{\partial\tau}F} > 0. \quad (70)$$

By (5b) we get that $1/\lambda$ depends on $\psi$ only through $\tau$, and thus by inspection it follows that liquidity increases in $\psi$.

Section A.1 of the Internet Appendix shows that a unique equilibrium with information acquisition exists if instead of risk seekers we have risk-neutral rational traders and noise traders. In this equilibrium, $d\tau/d\psi > 0$, but liquidity decreases in $\psi$ if $\psi$ is sufficiently high.

**Observable prices and imperfect competition** Let $F_p(\tau, \psi)$ denote the left-hand side of (69). As with $F(\tau, \psi)$, $F_p(\tau, \psi)$ decreases in $\tau$ in equilibrium because $\tau$ is the unique positive solution of a cubic polynomial with a negative leading coefficient. By the implicit function theorem we obtain

$$\frac{d\tau}{d\psi} = -\frac{\frac{\partial}{\partial\psi}F_p}{\frac{\partial}{\partial\tau}F_p} = -\frac{N(N-2)\tau + (N-1)\tau_D}{\frac{\partial}{\partial\tau}F_p} > 0, \tag{71}$$

as long as $N > 1$ (this constraint is without loss of generality because no trade happens with only one trader). By (10b) we get that $1/\lambda = N\gamma$ is an increasing function of $\tau$, and thus liquidity increases in $\psi$.

Section A.2 of the Internet Appendix extends the results of Section A.1 to observable prices—if we replace the risk seekers with risk-neutral rational traders and noise traders, then $d\tau/d\psi > 0$ but $d(1/\lambda)/d\psi < 0$ if $\psi$ is sufficiently high. ■

# II Heterogenous risk preferences

**Proof of Theorem 4.** Given the price conjecture in (14), the utility of trader $a$ is

$$du_a = dX_a \left( \mathbb{E}\left[ D - P_{-a} \middle| dz_a \right] - \lambda \, dX_a \right) - \frac{1}{2}\delta(a) \, (dX_a)^2 \, \mathrm{Var}\left( D - P_{-a} \middle| dz_a \right), \tag{72}$$

where $P_{-a} = \lambda \int_{s \neq a} dX_s$ is the price function without the impact of trader $a$. The first-order condition over $dX_a$ gives

$$dX_a = \frac{\mathbb{E}\left[D - P_{-a} \middle| dz_a\right]}{2\lambda + \delta(a)\text{Var}\left(D - P_{-a} \middle| dz_a\right)}, \tag{73}$$

and thus, by the projection theorem and (13), it follows that

$$\beta(a) = \frac{\frac{\text{Cov}(D - P_{-a}, dz_a)}{\text{Var}(dz_a)}}{2\lambda + \delta(a)\left[\text{Var}\left(D - P_{-a}\right) - \frac{\text{Cov}^2(D - P_{-a}, dz_a)}{\text{Var}(dz_a)}\right]}. \tag{74}$$

The signal structure in (11) gives

$$\text{Var}\left(dz_a\right) = \tau_D^{-1}(da)^2 + \tau(a)^{-1}da = \left[\tau(a)^{-1} + O(da)\right] da. \tag{75}$$

Let $\mathbb{I}_{\{.\}}$ stand for the indicator function; $\mathbb{I}_{\{s \neq a\}}$ equals one if $s \neq a$ and zero if $s = a$. The price function without the impact of trader $a$ is

$$P_{-a} = \lambda \int_0^1 \mathbb{I}_{\{s \neq a\}} dX_s. \tag{76}$$

Moreover, due to (13), (14) implies

$$P_{-a} = \left(\lambda \int_0^1 \mathbb{I}_{\{s \neq a\}} \beta(s) ds\right) D + \lambda \int_0^1 \mathbb{I}_{\{s \neq a\}} \beta(s) \sqrt{\tau(s)^{-1}} dB_s, \tag{77}$$

which further implies

$$\mathrm{Cov}\left(D-P_{-a},dz_a\right)=\tau_D^{-1}\left(1-\lambda\int_0^1\mathbb{I}_{\{s\neq a\}}\beta(s)ds\right)da=\tau_D^{-1}\left(1-\lambda\int_0^1\beta(s)ds+O(ds)\right)da \tag{78}$$

and

$$\begin{aligned}\mathrm{Var}\left(D-P_{-a}\right)&=\tau_D^{-1}\left(1-\lambda\int_0^1\mathbb{I}_{\{s\neq a\}}\beta(s)ds\right)^2+\lambda^2\int_0^1\mathbb{I}_{\{s\neq a\}}\beta^2(s)\tau(s)^{-1}ds\\&=\tau_D^{-1}\left(1-\lambda\int_0^1\beta(s)ds\right)^2+\lambda^2\int_0^1\beta^2(s)\tau(s)^{-1}ds+O(ds).\end{aligned} \tag{79}$$

Equation (74) now becomes

$$\begin{aligned}\beta(a)&=\frac{\mathrm{Cov}\left(D-P_{-a},dz_a\right)}{\left\{2\lambda+\delta(a)\mathrm{Var}\left(D-P_{-a}\right)\right\}\mathrm{Var}\left(dz_a\right)-\delta(a)\mathrm{Cov}^2\left(D-P_{-a},dz_a\right)}\\&=\frac{\tau_D^{-1}\left(1-\lambda\int_0^1\beta(s)ds+O(ds)\right)}{\left\{2\lambda+\delta(a)\mathrm{Var}\left(D-P_{-a}\right)\right\}\left[\tau(a)^{-1}+O(da)\right]-\delta(a)\tau_D^{-2}\left(\lambda\int_0^1\beta(s)ds+O(ds)\right)^2da}\\&=\frac{\tau_D^{-1}\left(1-\lambda\int_0^1\beta(s)ds+O(ds)\right)}{\left\{2\lambda+\delta(a)\left[\tau_D^{-1}\left(1-\lambda\int_0^1\beta(s)ds\right)^2+\lambda^2\int_0^1\beta^2(s)\tau(s)^{-1}ds\right]\right\}\tau(a)^{-1}+O(da)}.\end{aligned} \tag{80}$$

Sending $ds$ and $da$ to zero thus yields

$$\beta(a)=\frac{\tau(a)}{2\rho+\delta(a)\left[1-\lambda\int_0^1\beta(s)ds+\tau_D\frac{\lambda^2\int_0^1\beta^2(s)\tau(s)^{-1}ds}{1-\lambda\int_0^1\beta(s)ds}\right]}, \tag{81}$$

where the auxiliary term $\rho$ is

$$\rho=\frac{\lambda\tau_D}{1-\lambda\int_0^1\beta(s)ds}. \tag{82}$$

From (12) and (14) it follows that

$$\lambda = \frac{\mathrm{Cov}\left(D, \int_0^1 dX_s\right)}{\mathrm{Var}\left(\int_0^1 dX_s\right)} = \frac{\int_0^1 \beta(s)ds}{\left(\int_0^1 \beta(s)ds\right)^2 + \tau_D \int_0^1 \frac{\beta^2(s)}{\tau(s)}ds}, \tag{83}$$

and thus

$$1 - \lambda \int_0^1 \beta(s)ds = \frac{\tau_D \int_0^1 \frac{\beta^2(s)}{\tau(s)}ds}{\left(\int_0^1 \beta(s)ds\right)^2 + \tau_D \int_0^1 \frac{\beta^2(s)}{\tau(s)}ds}, \tag{84}$$

which, together with (83), gives

$$\tau_D \frac{\lambda^2 \int_0^1 \beta^2(s)\tau(s)^{-1}ds}{1 - \lambda \int_0^1 \beta(s)ds} = \lambda \int_0^1 \beta(s)ds. \tag{85}$$

Using (85) in (81) proves (18a). Equation (18b) follows directly from (82). Substituting (83) and (85) into (82) yields

$$\rho = \frac{\int_0^1 \beta(s)ds}{\int_0^1 \beta^2(s)\tau(s)^{-1}ds}. \tag{86}$$

Using (18a) to express the integrals in (86) proves (18c). From (86) it also follows that $\rho$ and $\int_0^1 \beta(s)ds$ must have the same sign. Finally, the second-order condition of trader $a$ is

$$-\left[2\lambda + \delta(a)\mathrm{Var}\left(D - P_{-a}\middle| dz_a\right)\right] = -\left[\frac{2\lambda}{\mathrm{Var}\left(D - P_{-a}\middle| dz_a\right)} + \delta(a)\right]\mathrm{Var}\left(D - P_{-a}\middle| dz_a\right) \tag{87}$$

The second-order condition of trader $a$ is thus satisfied if and only if

$$\frac{2\lambda}{\mathrm{Var}\left(D - P_{-a}\middle| dz_a\right)} + \delta(a) > 0. \tag{88}$$

Using the moments in (75), (78), and (79) to calculate the conditional variance shows, after some algebra, that

$$\lim_{da \to 0} \frac{\lambda}{\operatorname{Var}\left(D - P_{-a} \middle| dz_a\right)} = \rho, \tag{89}$$

and thus as $da \to 0$ the left-hand side of (88) equals $2\rho + \delta(a)$. ■

**Proof of Corollary 5.** By Itô's "box" calculus, the demand diversity is

$$\mathcal{V} = \int_0^1 (dX_a)^2 = \int_0^1 \frac{\beta^2(a)}{\tau(a)} da, \tag{90}$$

and the variance of the noisy part of the aggregate demand is

$$\mathcal{A} = \operatorname{Var}\left(\int_0^1 \beta(a)\sqrt{\tau(a)^{-1}} dB_a\right) = \int_0^1 \frac{\beta^2(a)}{\tau(a)} da = \frac{\int_0^1 \beta(a) da}{\rho}, \tag{91}$$

where the second equality follows from (86). It follows that $\mathcal{V}$ equals the variance of the aggregate price noise, and that (22a) holds.

By an embedded Random-Walk argument and elementary properties of random variables it can be shown that

$$\mathcal{E}^2 = \max\left\{\int_0^1 \left(dX_a^+\right)^2, \int_0^1 \left(dX_a^-\right)^2\right\} = \frac{1}{2}\int_0^1 (dX_a)^2 + \frac{1}{2}\left|\int_0^1 |dX_a| dX_a\right|. \tag{92}$$

(See Section G, and Theorem 17 in particular, for details.) The lower bound in (22b) now follows immediately, and the upper bound in (22b) follows by the triangle inequality. ■

**Proof of Theorem 6.** Assume that the second-order condition of all traders is strictly satisfied. There are three potential cases to consider for equilibrium, $\rho < 0$, $\rho = 0$, and $\rho > 0$. I show, by contradiction for each case, that no equilibrium exists if $\delta(a)$ is non-negative for all $a$.

Suppose that $\rho < 0$. Because $\tau > 0$, Equation (18c) implies that the second-order condition for a set of traders of positive measure must be violated.

Suppose that $\rho = 0$. If $\delta(a) = 0$ for at least some traders $a$ of non-zero measure, then the second-order condition is weakly violated for those traders. If, instead, $\delta(a) > 0$ for all $a$, Equation (18c) implies that

$$0 = \int_0^1 \frac{\tau(a)}{\delta(a)} da, \tag{93}$$

which is not possible.

Suppose that $\rho > 0$. We have two distinct cases, $\delta(a) = 0$ for all $a$, and $\delta(a) \geq 0$ with $\delta(a) > 0$ for at least some $a$ of non-zero measure. If $\delta(a) = 0$ for all $a$, then (18c) implies

$$\rho = \frac{\frac{\int_0^1 \tau(a)da}{2\rho}}{\frac{\int_0^1 \tau(a)da}{4\rho^2}} = 2\rho, \tag{94}$$

which is a contradiction. If $\delta(a) \geq 0$, then because $\rho > 0$ we get that

$$0 < \frac{\rho}{2\rho + \delta(a)} \leq 1. \tag{95}$$

It follows that

$$0 < \rho \frac{\tau(a)}{[2\rho + \delta(a)]^2} \leq \frac{\tau(a)}{2\rho + \delta(a)}. \tag{96}$$

Integrating both sides we obtain, because the inequality in (96) is strict for at least some $a$ of non-zero measure, that

$$\rho \int_0^1 \frac{\tau(a)}{[2\rho + \delta(a)]^2} da < \int_0^1 \frac{\tau(a)}{2\rho + \delta(a)} da, \tag{97}$$

which contradicts (18c). ■

**Proof of Proposition 7.** For a constant precision function, Equation (18c) gives

$$\rho = (2\rho - \chi\phi)\left[2\rho + \chi(1-\phi)\right]\frac{1}{\chi}\log\left(1 + \frac{\chi}{2\rho - \chi\phi}\right). \tag{98}$$

The second-order condition of the most risk-seeking traders is satisfied if $2\rho - \chi\phi > 0$, or, equivalently, if $\rho > \chi\phi/2$. Because $2\rho + \chi(1-\phi) > 2\rho - \chi\phi$, if $\rho > \chi\phi/2$ holds then the logarithm on the right-hand side of (98) is well defined. Thus, to prove that (98) has a solution which strictly satisfies the second-order condition of all traders, it suffices to look for a root in $(\chi\phi/2, \infty)$.

Equation (98) is a fixed-point mapping in $\rho$. The left-hand side of (98) is a 45° degree line; I prove that the right-hand side crosses the 45° degree line at least once from below.

At $\rho = \chi\phi/2$ the right-hand side of (98) lies below its left-hand side because

$$\lim_{\rho \to \frac{\chi\phi}{2}} (2\rho - \chi\phi)\left[2\rho + \chi(1-\phi)\right]\frac{1}{\chi}\log\left(1 + \frac{\chi}{2\rho - \chi\phi}\right) = 0 < \frac{\chi\phi}{2} \tag{99}$$

At $\rho = \chi\phi$ the right-hand side of (98) lies above its left-hand side because

$$(2\rho - \chi\phi)\left[2\rho + \chi(1-\phi)\right]\frac{1}{\chi}\log\left(1 + \frac{\chi}{2\rho - \chi\phi}\right)\Bigg|_{\rho=\chi\phi} = \chi\phi(1+\phi)\log\left(1 + \frac{1}{\phi}\right) > \chi\phi, \tag{100}$$

where the inequality follows due to the fundamental logarithmic inequality

$$\log\left(1 + \frac{1}{\phi}\right) > \frac{1}{1+\phi} \tag{101}$$

The right-hand of (98) thus crosses its left-hand side somewhere in the interval $(\chi\phi/2, \chi\phi)$, and a solution to (98) thus exists. ■

**Lemma II.1**
$$\frac{1}{\tau_D}\left(1-\lambda\int_0^1 \beta(s)ds\right)=\frac{\lambda}{\rho}=\frac{1}{\rho\int_0^1 \beta(s)ds+\tau_D}. \tag{102}$$

**Proof.** The result follows by combining (84) and (86). ■

**Proof of Lemma 8.** Combining (72) with (73) gives

$$du\left(\pi_a; dz_a\right)=\frac{1}{2}\mathbb{E}\left[D-P_{-a}\middle|dz_a\right]dX_a. \tag{103}$$

For $s\neq a$

$$\mathbb{E}\left[dX_s\middle|dz_a\right]=\mathbb{E}\left[D\middle|dz_a\right]\beta(s)ds, \tag{104}$$

and therefore

$$du\left(\pi_a; dz_a\right)=\frac{1}{2}\left(1-\lambda\int_0^1 \mathbb{I}_{\{s\neq a\}}\beta(s)ds\right)\mathbb{E}\left[D\middle|dz_a\right]dX_a. \tag{105}$$

Taking expectations thus implies

$$\begin{aligned}\mathbb{E}\left[du\left(\pi_a; dz_a\right)\right]&=\frac{1}{2}\left(1-\lambda\int_0^1 \mathbb{I}_{\{s\neq a\}}\beta(s)ds\right)\beta(a)\mathbb{E}\left[\mathbb{E}\left[D\middle|dz_a\right]dz_a\right]\\&=\frac{1}{2}\left(1-\lambda\int_0^1 \mathbb{I}_{\{s\neq a\}}\beta(s)ds\right)\beta(a)\frac{\tau(a)}{\tau(a)da+\tau_D}\mathbb{E}\left[(dz_a)^2\right]\\&=\frac{1}{2\tau_D}\left(1-\lambda\int_0^1 \beta(s)ds\right)\beta(a)da+O\left(da^2\right).\end{aligned} \tag{106}$$

Ignoring terms of order $da^2$ and higher, the first-order condition of (26) is

$$\frac{1}{2\tau_D}\left(1-\lambda\int_0^1 \beta(s)ds\right)\frac{1}{\delta(a)+2\rho}da=\frac{\tau(a)}{2\psi(a)}da. \tag{107}$$

Substituting (102) of Lemma II.1 into (107) proves (27), while omitted algebra shows that every trader is strictly better off by acquiring information. ■

**Proof of Theorem 9.** Equations (28a) and (28b) and the restriction on the sign of $\rho$ and $\int_0^1 \beta(s)ds$ follow from Theorem 4 and Lemma 8. ■

**Proof of Theorem 10.** There are three potential cases to consider for equilibrium, $\rho < 0$, $\rho = 0$, and $\rho > 0$. I show, by contradiction for each case, that no equilibrium exists if $\delta(a)$ is non-negative for all $a$.

Suppose that $\rho < 0$. Writing (28b) as a quadratic polynomial of $\int_0^1 \beta(s)ds$ then implies that there are two strictly positive solutions for $\int_0^1 \beta(s)ds$, which contradicts the restriction of Theorem 9 that $\int_0^1 \beta(s)ds$ and $\rho$ must have the same sign.

Suppose that $\rho = 0$. This implies that (28a) becomes

$$0 = \int_0^1 \frac{\psi(s)}{[\delta(s)]^2} ds, \tag{108}$$

which is not possible.

Suppose that $\rho > 0$. We have two distinct cases, $\delta(a) = 0$ for all $a$, and $\delta(a) \geq 0$ with $\delta(a) > 0$ for at least some $a$ of non-zero measure. If $\delta(a) = 0$ for all $a$, then (28a) becomes

$$\rho \frac{\int_0^1 \psi(s)ds}{8\rho^3} = \frac{\int_0^1 \psi(s)ds}{4\rho^2}, \tag{109}$$

which is a contradiction. If $\delta(a) \geq 0$, then because $\rho > 0$ we get that

$$0 < \frac{\rho}{2\rho + \delta(a)} \leq 1. \tag{110}$$

It follows that

$$0 < \rho \frac{\psi(a)}{[2\rho + \delta(a)]^3} \leq \frac{\psi(a)}{[2\rho + \delta(a)]^2}. \tag{111}$$

Integrating both sides we obtain, because the inequality in (111) is strict for at least some $a$

of non-zero measure, that

$$\rho \int_0^1 \frac{\psi(s)}{[2\rho + \delta(s)]^3} ds < \int_0^1 \frac{\psi(s)}{[2\rho + \delta(s)]^2} ds, \tag{112}$$

which contradicts (28a). ■

**Lemma II.2** *For the example of section 3.3.1, the equilibrium conditions of* (28a) *and* (28b) *are*

$$2\rho^2 - 3\left(\phi - \frac{1}{2}\right)\chi\rho - \phi(1-\phi)\chi^2 = 0 \tag{113a}$$

*and*

$$\rho\left(\int_0^1 \beta(s)ds\right)^2 + \tau_D\left(\int_0^1 \beta(s)ds\right) - \frac{\psi}{[2\rho - \phi\chi]\,[2\rho + (1-\phi)\chi]} = 0. \tag{113b}$$

**Proof.** The specification in (24) implies that the equilibrium condition in (28a) becomes

$$\rho \int_0^1 \frac{1}{[2\rho - \chi\phi + \chi s]^3} ds = \int_0^1 \frac{1}{[2\rho - \chi\phi + \chi s]^2} ds. \tag{114}$$

After integrating both sides we obtain

$$\rho \frac{4\rho + (1-2\phi)\chi}{2\,[2\rho - \phi\chi]^2\,[2\rho + (1-\phi)\chi]^2} = \frac{1}{[2\rho - \phi\chi]\,[2\rho + (1-\phi)\chi]}, \tag{115}$$

and after rearranging and simplifying we obtain (113a). Given a solution $\rho_\star$ of (113a), (28b) gives (113b) evaluated at $\rho = \rho_\star$. ■

**Proof of Proposition 11.** By Lemma II.2, the equilibrium with the preference specification in (24) is given in system (113). Let $G\left(\rho, \int_0^1 \beta(s)ds; \phi\right)$ stand for the left-hand side of (113a) and $J\left(\rho, \int_0^1 \beta(s)ds; \phi\right)$ stand for the left-hand side of (113b). There are two cases, $\phi \geq 1/2$ and $\phi < 1/2$.

If $\phi \geq 1/2$, the sequence of signs of the coefficients of (113a) is positive, nonpositive, and negative. If $\phi < 1/2$, the sequence of signs of the coefficients of (113a) is positive, positive, and negative. In either case, Equation (113a) has a unique positive solution for $\rho$ by Descartes's rule of signs. Let $\rho_\star$ denote this root. Because the leading coefficient in (113a) is positive, this further implies that $\phi\chi/2 < \rho_\star$, if and only if

$$G\left(\frac{\phi\chi}{2};\phi\right) < 0. \tag{116}$$

We have

$$G\left(\frac{\phi\chi}{2};\phi\right) = -\frac{\phi\chi^2}{4}, \tag{117}$$

which proves (116), and that

$$2\rho_\star - \phi\chi > 0 \tag{118}$$

holds. This proves that the second-order condition of the traders with the most negative risk-preference coefficient is satisfied. It now follows that the second-order condition of every trader is satisfied.

Given the unique solution $\rho_\star$ of (113a), condition (113b) is a quadratic polynomial in $\int_0^1 \beta(s)ds$, with positive coefficients for the quadratic term and the linear term. The constant term in is negative due to that $2\rho_\star - \phi\chi > 0$, which also implies that $2\rho_\star + (1-\phi)\chi > 0$. It follows by Descartes's rule of signs that (113b) has a unique positive solution. This proves that a unique equilibrium exists.

By (113a) of Lemma II.2, $\rho$ is not affected by $\psi$ (this also follows from (28a) of Theorem 9 with $\psi(s) = \psi$ for all $s$.) Applying the Implicit Function Theorem on (113b) of Lemma II.2 we obtain that $\int_0^1 \beta(s)ds$ increases in $\psi$, and thus by (18b) of Theorem 4 so does $\lambda^{-1}$.

For the proof of the claim with noise traders and weakly risk-averse rational trader, see Section F of the Internet Appedix. ■

**A note on the proof of Corollary 12.** The proof is a straightforward, but laborious, application of the two-dimensional version of the Implicit Function Theorem. The complete proof is in Section C of the internet Appendix. ■

# III Extension to limit orders

## III.1 Summary of the heterogeneous market with limit orders

Similarly to Kyle (1989), we can write the price function by separating out the impact of a given trader $a$, as

$$P = P_{-a} + \lambda_{-a} dX_a \tag{119}$$

so that $\lambda_{-a}$ is the slope and $P_{-a}$ is the intercept of a residual supply curve. (See Section D of the Internet Appendix for details.) The utility of trader $a$ is the same as in Section 3.1, with the only difference that there is now an extra signal. The first-order condition gives, after using the residual demand schedule, that

$$dX_a = \frac{\mathbb{E}\left[D - P \middle| dz_a, d\zeta_a\right]}{\lambda_{-a} + \delta(a)\mathrm{Var}\left(D - P_{-a} \middle| dz_a, d\zeta_a\right)}. \tag{120}$$

Writing out the conditional moments, comparing the resulting expression to the demand strategy in (31), and treating each trader as small ($da \to 0$) yields the equilibrium in Theorem 13. A detailed proof is in Section D of the Internet Appendix.

# IV Figures for the main text

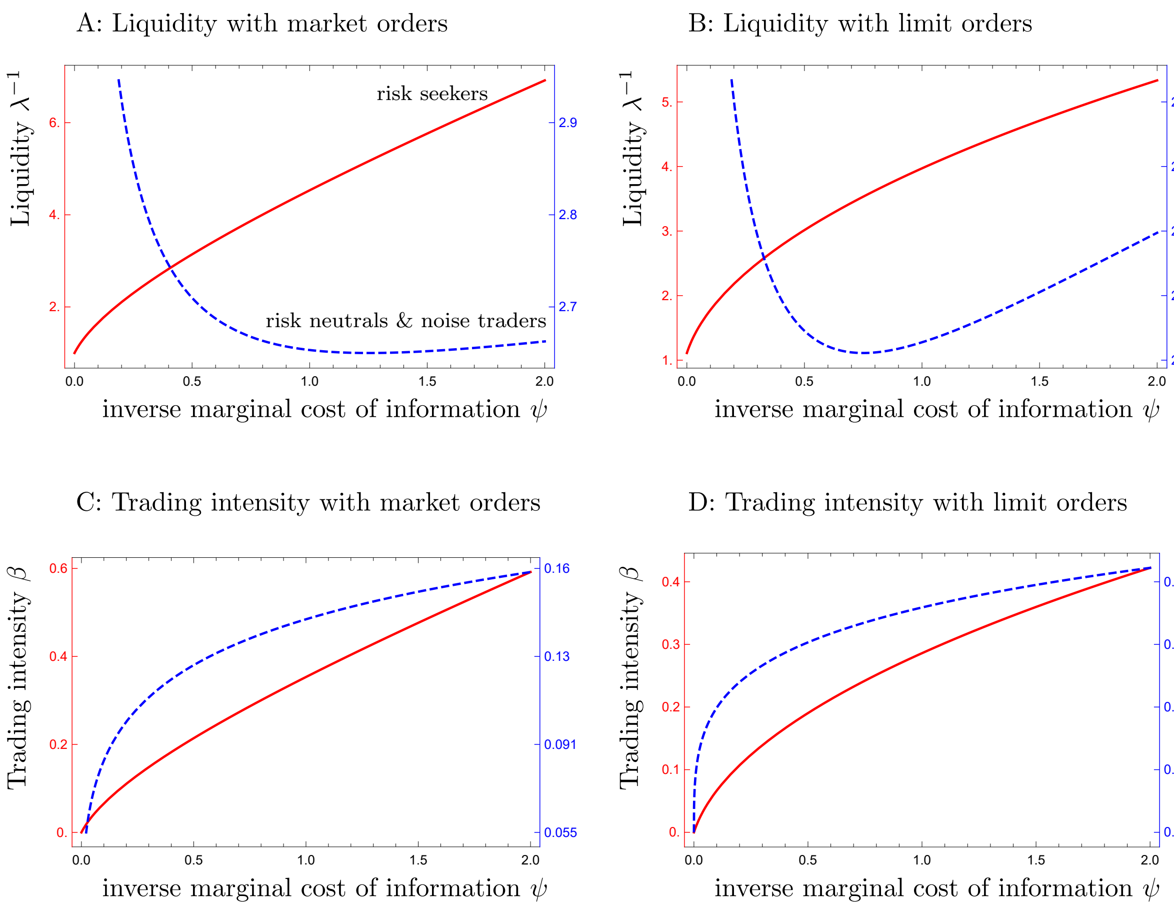


Figure 1: Liquidity (top row) and trading intensity (bottom row) as a function of how cheap it is to acquire information in economies with market orders (left column) and with limit orders (right column). In each panel, the solid red curves show the risk seeking economy, with scales on the left vertical axis; the blue dash-dotted blue curves show the economy with noise traders, with scales on the right vertical axis. For the plots with risk seekers the risk preference parameter is $\delta = -1$, while for the plots with noise traders the rational traders are risk neutral and the precision of the noise traders' demand is $\tau_\theta = 1$. For all plots, the precision of the dividend is $\tau_D = 1$, and the number of rational traders is $N = 10$.

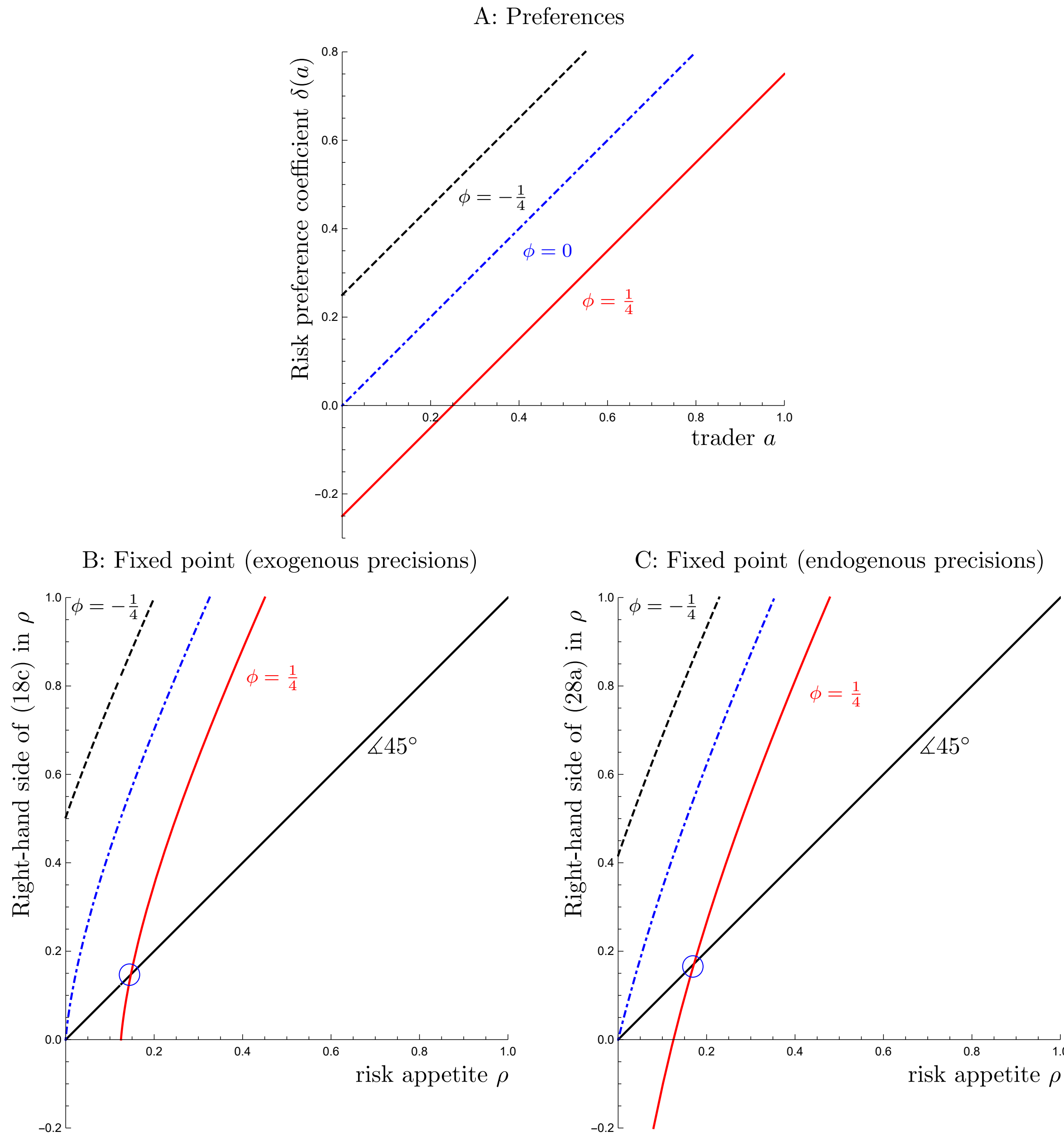


Figure 2: The risk-preference coefficient and the associated equilibrium conditions for three different risk-preference settings. Panel A shows the risk preference coefficient $\delta(a)$ as a function of trader $a$, for the settings $\phi = -1/4$ (dashed black), $\phi = 0$ (dash-dotted blue), and $\phi = 1/4$ (solid red). Panel B shows the right-hand side of the equilibrium condition in (18c) as a function of risk appetite $\rho$ for each of the risk-preference settings with exogenous and homogeneous precisions, drawn in the same styles as in Panel A. Panel C shows the same type of picture as Panel B, but with endogenous precisions; the equilibrium condition is (28a). For an equilibrium to exist, the curves in Panels B and C must intersect the 45° line, which is drawn in solid black. We do not have an equilibrium with positive $\rho$ for $\phi \leq 0$, i.e. when there are no risk seekers. For $\phi = 1/4$ the solid red curve intersects the 45° line at the point indicated by the blue circle—the corresponding equilibrium solution is $\rho_* \approx 0.147$ in Panel B and $\rho_* \approx 0.172$ in Panel C. For Panel B the precision is $\tau(a) = 1$ for all $a$, and for Panel C the inverse of the marginal cost of information is $\psi(a) = 1$ for all $a$. For all graphs, the preference spread is $\chi = 1$ and the precision of the dividend is $\tau_D = 1$.

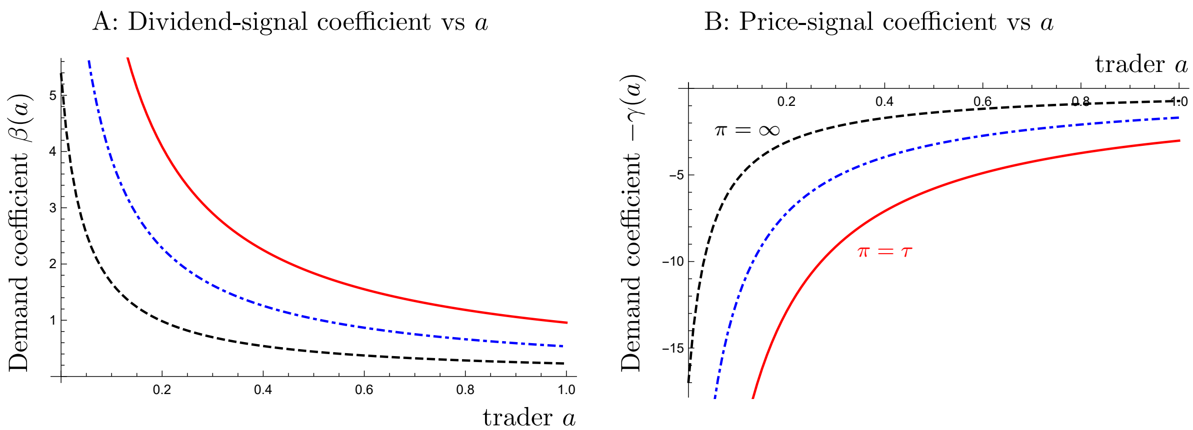


Figure 3: Demand coefficients of dividend signals (Panel A) and price signals (Panel B) for the noisy limit-order model as a function of trader a, for the same risk preferences as in Figure 2 with $\phi = 1/4$. The three curves correspond to equilibria where each trader pays increasing amounts of attention to prices. The solid red curves set $\pi(a) = \tau(a)$ for all $a$, the dash-dotted blue curves set $\pi(a) = 4\tau(a)$ for all $a$, while the dashed black curves have fully observable prices, $\pi(a) = \infty$. The precision of the dividend is $\tau_D = 1$, the dividend precision is $\tau(a) = 1$ for all traders $a$, and the preference spread is $\chi = 1$.

# Internet Appendix

## A note on the contents

This Internet Appendix provides supplementary material for the main paper. It has seven sections, which are ordered to reflect the order of presentation in the main text. In terms of complementing the exposition of the argument in the main text, the two most prominent Sections are B and G.

Section A derives the noise-trading models used in the comparison in Corollary 3.

Section B solves an equilibrium with one risk seeker, many risk-neutral traders, and observable prices. This is similar to Kyle (1989), but without noise traders. This model is meant as a robustness check, exhibiting the loss of tractability associated with heterogeneous versions of the finite limit-order market, while maintaining the main results.

Section C contains detailed proofs of the comparative statics in Section 3.4.

Section D contains detailed proofs for the model in Section 3.5.

Section E derives sufficient conditions for existence and uniqueness of equilibrium with risk preferences which are non-linear functions of the traders' location in the unit interval.

Section F derives a model with noise traders and homogeneous risk aversion. This section is meant to show two things. First, that noise traders can be readily accommodated in a large-market similar to that of Section 3. Second, that the empirical distinction in Corollary 3 continues to hold between the heterogeneous market with partial risk seeking and a market where the risk seekers are replaced by noise traders, irrespective of risk aversion.

Section G derives the heterogeneous market as a monopolistic-competition limit of a finite economy. Alternatively, this section can be thought of as a proof that the continuum model "embeds" a finite economy with imperfect competition. An earlier version of the paper, titled "Risk Seeking As Noise Provision," used this limit for exposition of the results.

# A Models with noise traders, risk-neutral traders, and information acquisition (Corollary 3)

## A.1 Unobservable prices and competitive market makers

The model is the same as that in Section 2.1, with two differences: first, the rational traders are risk-neutral, and second, there are noise traders. The aggregate demand of the noise traders is $\theta \sim \mathcal{N}(0, \tau_\theta)$, and it is independent of $D$ and $\varepsilon_n$, $n = 1, \ldots, N$.

The demand conjecture for the rational traders is the same as in (4a), but the market maker's pricing rule is now

$$P = \mathbb{E}\left[D \middle| \sum_{n=1}^{N} X_n + \theta\right], \tag{A.1a}$$

and the price conjecture is

$$P = \lambda \left(\sum_{n=1}^{N} X_n + \theta\right). \tag{A.1b}$$

The profit for rational trader $n$ is $\pi_n = X_n(D - P)$, and his utility is $\mathbb{E}\left[\pi_n \middle| s_n\right]$. This implies that trader $n$'s optimal demand is

$$X_n = \frac{\mathbb{E}\left[D - P_{-n} \middle| s_n\right]}{2\lambda}. \tag{A.2}$$

Comparing (4a) with (A.2) we obtain

$$2\lambda\beta_n = \frac{\tau_n}{\tau_n + \tau_D}\left(1 - \lambda \sum_{\substack{i=1 \\ i \neq n}}^{N} \beta_i\right), \tag{A.3}$$

from which it follows that

$$\lambda\beta_n = \frac{\tau_n}{\tau_n + 2\tau_D}\left(1 - \sum_{i=1}^{N}\lambda\beta_i\right), \tag{A.4}$$

and that

$$\sum_{n=1}^{N}\lambda\beta_n = \left(\sum_{n=1}^{N}\frac{\tau_n}{\tau_n + 2\tau_D}\right)\left(1 - \sum_{i=1}^{N}\lambda\beta_i\right). \tag{A.5}$$

Solving (A.5) we get

$$\sum_{n=1}^{N}\lambda\beta_n = \left(\sum_{n=1}^{N}\frac{\tau_n}{\tau_n + 2\tau_D}\right)\left(1 + \sum_{n=1}^{N}\frac{\tau_n}{\tau_n + 2\tau_D}\right)^{-1}, \tag{A.6}$$

and plugging into (A.4) we obtain

$$\lambda\beta_n = \frac{\tau_n}{\tau_n + 2\tau_D}\left(1 + \sum_{i=1}^{N}\frac{\tau_i}{\tau_i + 2\tau_D}\right)^{-1}. \tag{A.7}$$

By (A.1) we have

$$\lambda = \frac{\operatorname{Cov}\left(D, \sum_{i=1}^{N} X_i + \theta\right)}{\operatorname{Var}\left(\sum_{i=1}^{N} X_i + \theta\right)} = \frac{\frac{1}{\tau_D}\sum_{i=1}^{N}\beta_i}{\frac{1}{\tau_D}\left(\sum_{i=1}^{N}\beta_i\right)^2 + \sum_{i=1}^{N}\frac{\beta_i^2}{\tau_i} + \frac{1}{\tau_\theta}}, \tag{A.8}$$

which implies

$$\lambda^2 = \tau_\theta\left(\frac{1}{\tau_D}\sum_{i=1}^{N}\lambda\beta_i - \frac{1}{\tau_D}\left(\sum_{i=1}^{N}\lambda\beta_i\right)^2 - \sum_{i=1}^{N}\frac{(\lambda\beta_i)^2}{\tau_i}\right). \tag{A.9}$$

Using (A.7) yields

$$\lambda^2 = \tau_\theta\frac{\sum_{i=1}^{N}\frac{\tau_i(\tau_i+\tau_D)}{\tau_D(\tau_i+2\tau_D)^2}}{\left(1 + \sum_{i=1}^{N}\frac{\tau_i}{\tau_i+2\tau_D}\right)^2}. \tag{A.10}$$

By the law of iterated expectations, (A.1b), (A.2) and (A.10) it follows that

$$\mathbb{E}\left[\pi_n\right] = \lambda\mathbb{E}\left[X_n^2\right] = \frac{(\lambda\beta_n)^2}{\lambda}\frac{\tau_n+\tau_D}{\tau_n\tau_D} = \frac{\frac{\tau_n(\tau_n+\tau_D)}{\tau_D(\tau_n+2\tau_D)^2}}{\sqrt{\tau_\theta}\sqrt{\sum_{i=1}^N \frac{\tau_i(\tau_i+\tau_D)}{\tau_D(\tau_i+2\tau_D)^2}}}\left(1+\sum_{i=1}^N \frac{\tau_i}{\tau_i+2\tau_D}\right)^{-1}. \tag{A.11}$$

With the same cost function as for Proposition 1, taking the first-order condition with respect to $\tau_n$ and then setting $\tau_n = \tau$ gives, after squaring both sides, that

$$\begin{aligned}&\tau_\theta N^3\tau^3\left(\tau+\tau_D\right)\left(\tau+2\tau_D\right)^2\left[(N+1)\tau+2\tau_D\right]^4 \\ &\quad -\psi^2\tau_D\left[\left(6N^2-N-3\right)\tau^2+2\left(2N^2+5N-4\right)\tau_D\tau+4(2N-1)\tau_D^2\right]^2=0\end{aligned} \tag{A.12}$$

Condition (A.12) is the equilibrium condition for information acquisition with symmetric precisions. It is a tenth-order polynomial in $\tau$, and it can be shown that is has a unique root by Descartes' rule of signs. (By inspection, the positive term contributes positive powers in descending orders of ten to three. The negative term contributes negative powers in descending orders of four to zero. In particular, the constant is always negative, which guarantees existence, and the polynomial contains powers from both of the two terms only for powers four and three. If $N$ is large enough, those powers are both positive, while if $\psi$ is large enough, those powers are both negative. The owers of orders two, one and zero are always negative, and thus we have a unique positive solution.)

Let $H(\tau,\psi)$ denote the left-hand side of (A.12). Noting that $H(\tau,\psi)$ is increasing in $\tau$ in equilibrium (the equilibrium condition in (A.12) has a unique positive root in $\tau$ , and because its leading coefficient is positive it must cross zero from below), we obtain

$$\frac{d\tau}{d\psi} = -\frac{\frac{\partial H}{\partial\psi}}{\frac{\partial H}{\partial\tau}} = \frac{2\psi\tau_D\left[\left(6N^2-N-3\right)\tau^2+2\left(2N^2+5N-4\right)\tau_D\tau+4(2N-1)\tau_D^2\right]^2}{\frac{\partial H}{\partial\tau}} > 0. \tag{A.13}$$

From (A.10) we have that in equilibrium

$$\frac{1}{\lambda^2} = \frac{\tau_D}{\tau_\theta N} \frac{\left[(N+1)\tau + 2\tau_D\right]^2}{\tau\left(\tau + \tau_D\right)}. \tag{A.14}$$

and therefore liquidity depends on $\psi$ only though its effect on $\tau$, so that

$$\frac{d}{d\psi}\left(\frac{1}{\lambda^2}\right) = \frac{d}{d\tau}\left(\frac{1}{\lambda^2}\right)\frac{d\tau}{d\psi} \tag{A.15}$$

where by (A.14), we have

$$\frac{d}{d\tau}\left(\frac{1}{\lambda^2}\right) = \frac{\tau_D^2}{\tau_\theta N} \frac{\left[(N+1)\tau + 2\tau_D\right]}{\tau\left(\tau + \tau_D\right)}\left[(N-3)\tau - 2\tau_D\right]. \tag{A.16}$$

If $N \leq 3$, the above is always negative and thus liquidity decreases in $\psi$. If $N > 3$, whether liquidity is increasing or decreasing in $\psi$ thus boils down to the sign of

$$(N-3)\tau - 2\tau_D \tag{A.17}$$

in equilibrium.

Let $\tau_c$ be the value of $\tau$ for which (A.17) is zero, that is, $\tau_c = 2\tau_D/(N-3)$. It suffices to derive conditions for $\tau < \tau_c$, because then

$$(N-3)\tau - 2\tau_D < 0, \tag{A.18}$$

which implies that liquidity decreases in $\tau$ (and thus liquidity also decreases in $\psi$.) To wit, because the equilibrium condition in (A.12) has a unique positive root in $\tau$, it suffices to check when $H(\tau_c, \psi) > 0$, which implies that $\tau_c > \tau$ in equilibrium. We have

$$H(\tau_c, \psi) = \frac{256\tau_D^5 N^2(N-1)^4}{(N-3)^{10}} \left[(N-3)^6\psi^2 - 32\tau_\theta\tau_D^5 N(N-1)(N-2)^2\right], \tag{A.19}$$

which is positive for large enough $\psi$ (and negative for small enough $\psi$).

## A.2 Observable prices and imperfect competition

The model is the same as in Section 2.2, but now the traders are risk neutral ($\delta = 0$) and there are noise traders, whose aggregate demand is $\theta \sim \mathcal{N}(0, \tau_\theta)$, independently of $D$ and $\varepsilon_n$, $n = 1, \ldots, N$.

The derivation follows along the lines of the proof of Proposition 2; I thus highlight the differences. The market clears stochastically,

$$\sum_{n=1}^{N} X_n + \theta = 0, \tag{A.20}$$

from which we obtain

$$P = \lambda \left( D \sum_{n=1}^{N} \beta_n + \sum_{k=1}^{N} \beta_k \varepsilon_k + \theta \right). \tag{A.21}$$

The definitions of $\lambda$ and $\lambda_{-n}$ are algebraically the same as before. Following Kyle (1989), $P_{-n}$ can be written as

$$P_{-n} = \lambda_{-n} \left[ D \sum_{\substack{k=1 \\ k \neq n}}^{N} \beta_k + \sum_{\substack{k=1 \\ k \neq n}}^{N} \beta_k \varepsilon_k + \theta \right]. \tag{A.22}$$

As before, by the projection theorem

$$\mathbb{E}\left[D \middle| s_n, P_{-n}\right] = b_n s_n + c_n P_{-n} \tag{A.23}$$

but now

$$b_n = \frac{\tau_n}{\tau_D + \tau_n + \frac{\left(\sum_{\substack{k=1\\k\neq n}}^N \beta_k\right)^2}{\sum_{\substack{k=1\\k\neq n}}^N \frac{\beta_k^2}{\tau_k} + \frac{1}{\tau_\theta}}}, \tag{A.24a}$$

$$c_n = \sum_{\substack{k=1\\k\neq n}}^N \gamma_k \frac{\frac{\sum_{\substack{k=1\\k\neq n}}^N \beta_k}{\sum_{\substack{k=1\\k\neq n}}^N \frac{\beta_k^2}{\tau_k} + \frac{1}{\tau_\theta}}}{\tau_D + \tau_n + \frac{\left(\sum_{\substack{k=1\\k\neq n}}^N \beta_k\right)^2}{\sum_{\substack{k=1\\k\neq n}}^N \frac{\beta_k^2}{\tau_k} + \frac{1}{\tau_\theta}}}, \tag{A.24b}$$

and

$$\operatorname{Var}\left(D \middle| s_n, P_{-n}\right) = \frac{1}{\tau_D + \tau_n + \frac{\left(\sum_{\substack{k=1\\k\neq n}}^N \beta_k\right)^2}{\sum_{\substack{k=1\\k\neq n}}^N \frac{\beta_k^2}{\tau_k} + \frac{1}{\tau_\theta}}} \tag{A.24c}$$

Matching coefficients in the demand conjecture and the demand functions implied by the equilibrium we obtain

$$\beta_n = \frac{b_n}{\lambda_{-n}\left(1 + c_n\right)} \tag{A.25a}$$

and

$$\gamma_n = \frac{\left(1 - c_n\right)}{\lambda_{-n}\left(1 + c_n\right)}. \tag{A.25b}$$

Under homogeneous precisions ($\tau_n = \tau$ for all $n$) we get

$$\beta = \sqrt{\tau}\sqrt{\frac{N-2}{\tau_\theta N(N-1)}} \tag{A.26}$$

and

$$\gamma = \frac{N\tau + 2\tau_D}{N\sqrt{\tau}}\sqrt{\frac{N-2}{\tau_\theta N(N-1)}}. \tag{A.27}$$

To justify the equilibrium with homogeneous precisions, suppose that each trader $n$ faces the information cost function in (3). His ex-ante utility is

$$\begin{aligned}
&\mathbb{E}\left[u\left(\pi_n; s_n, P_{-n}\right)\right] = \mathbb{E}\left[X_n^2\right]\lambda_{-n} \\
&= \left[\beta_n^2\left(\frac{1}{\tau_n} + \frac{1}{\tau_D}\right) - 2\beta_n\gamma_n \frac{\frac{\beta_n + \sum_{\substack{k=1\\k\neq n}}^N \beta_k}{\tau_D} + \frac{\beta_n}{\tau_n}}{\left(\sum_{\substack{k=1\\k\neq n}}^N \gamma_k + \gamma_n\right)} + \gamma_n^2 \frac{\frac{\left(\beta_n + \sum_{\substack{k=1\\k\neq n}}^N \beta_k\right)^2}{\tau_D} + \frac{\beta_n^2}{\tau_n} + \sum_{\substack{k=1\\k\neq n}}^N \frac{\beta_k^2}{\tau_k} + \frac{1}{\tau_\theta}}{\left(\sum_{\substack{k=1\\k\neq n}}^N \gamma_k + \gamma_n\right)^2}\right] \\
&\qquad \times \left[\frac{1}{\sum_{\substack{k=1\\k\neq n}}^N \gamma_k}\right]
\end{aligned} \tag{A.28}$$

Because each trader $k$ commits to $\beta_k$ and $\gamma_k$ after choosing $\tau_k$, it follows by inspection of (A.24) and (A.25), that if we fix $(\beta_k, \gamma_k, \tau_k)$ for $k \neq n$, changes in the utility in (A.28) happen only through $\tau_n$ (with $\beta_n$ and $\gamma_n$ as functions of $\tau_n$.) We may thus take the first-order condition of (A.28) holding $(\beta_k, \gamma_k, \tau_k)$ for $k \neq n$ as constants. Doing so, and setting $\tau_k = \tau$ we get

$$\frac{\sqrt{N-2}}{N\sqrt{\tau_\theta N(N-1)}} \frac{N(N-3)\left(\sqrt{\tau}\right)^2 + 2(N-1)\tau_D}{\sqrt{\tau}\left(N\left(\sqrt{\tau}\right)^2 + 2\tau_D\right)^2} = \frac{\left(\sqrt{\tau}\right)^2}{2\psi}. \tag{A.29}$$

Rearranging (A.29) we have

$$\sqrt{\tau_\theta N^7(N-1)}\left(\sqrt{\tau}\right)^7 + 4\sqrt{\tau_\theta N^5(N-1)}\tau_D\left(\sqrt{\tau}\right)^5 + 4\sqrt{\tau_\theta N^3(N-1)}\tau_D^2\left(\sqrt{\tau}\right)^3 \\ - 2\psi N(N-3)\sqrt{N-2}\left(\sqrt{\tau}\right)^2 - 4\psi\tau_D\sqrt{N-2} = 0. \quad \text{(A.30)}$$

By inspection it follows that the coefficients of the polynomial in (A.30) switch signs only once, between the cubic and the quadratic term and thus, by Descartes's rule of signs, (A.30) has a unique positive solution for $\sqrt{\tau}$. (Note that equilibrium exists only if $N > 3$.)

Let $H_p(\sqrt{\tau}, \psi)$ denote the left-hand side of (A.30). Noting that $H_p(\sqrt{\tau}, \psi)$ is increasing in $\sqrt{\tau}$ in equilibrium (the equilibrium condition in (A.30) has a unique positive root in $\sqrt{\tau}$, and because its leading coefficient is positive it must cross zero from below), we obtain

$$\frac{d\sqrt{\tau}}{d\psi} = -\frac{\frac{\partial H_p}{\partial \psi}}{\frac{\partial H_p}{\partial \sqrt{\tau}}} = \frac{2\sqrt{N-2}\left(N(N-3)\left(\sqrt{\tau}\right)^2 + 2\tau_D(N-1)\right)}{\frac{\partial H_p}{\partial \tau}} > 0, \quad \text{(A.31)}$$

because $N > 3$. By the chain rule, and because $\lambda^{-1} = N\gamma$,

$$\frac{d\lambda^{-1}}{d\psi} = \frac{d\lambda^{-1}}{d\sqrt{\tau}}\frac{d\sqrt{\tau}}{d\psi} = \sqrt{\frac{N-2}{\tau_\theta N(N-1)}}\frac{N\tau - 2\tau_D}{\tau}\frac{d\sqrt{\tau}}{d\psi}, \quad \text{(A.32)}$$

so that $d\lambda^{-1}/d\psi < 0$ if and only if

$$N\tau - 2\tau_D \quad \text{(A.33)}$$

is negative in equilibrium. Let $\tau_{pc}$ be the value of $\tau$ for which (A.17) is zero, that is, $\tau_{pc} = 2\tau_D/N$. Because the equilibrium condition in (A.30) has a unique positive root in $\sqrt{\tau}$ and it crosses zero from below, it suffices to show that $H_p\left(\sqrt{\tau_{pc}}, \psi\right) > 0$, which implies that

$\tau < \tau_{pc}$ in equilibrium. We have

$$H_p(\sqrt{\tau_{pc}}, \psi) = 8\tau_D \left[ 4\sqrt{2(N-1)\tau_\theta \tau_D^5} - \sqrt{(N-2)^3}\psi \right], \tag{A.34}$$

which is positive for small enough $\psi$ and negative for large enough $\psi$. This implies that $d\lambda^{-1}/d\psi < 0$ for small enough $\psi$, and $d\lambda^{-1}/d\psi > 0$ for large enough $\psi$.

## B One risk seeking trader, finite risk-neutral traders, observable prices, and imperfect competition

The model I present here is similar to Kyle (1989), but without noise traders. There are $N+1$ traders in total. Every trader $n = 1, \ldots, N+1$, observes the price and a signal as in (1), under the simplifying assumption that $\tau_n$ is the same for all $n$. The utility of trader $n$ is

$$u\left(\pi_n; s_n\right) = \mathbb{E}\left[\pi_n \middle| s_n\right] - \frac{1}{2}\delta_n \mathrm{Var}\left(\pi_n \middle| s_n\right), \tag{B.1}$$

where, as in the main text, the profit for trader $n$ is $\pi_n = X_n(D-P)$. The first $N$ traders are risk neutral (so that $\delta_n = 0$, $n = 1, \ldots, N$) and the $N$th trader likes risk ($\delta_n = \delta < 0$).

I assume that the price function is linear, and that the demand function of trader $n$ is

$$X_n = \beta_n s_n - \gamma_n P. \tag{B.2}$$

The market clears deterministically. We have, in particular, that

$$\sum_{n=1}^{N+1} X_n = 0, \tag{B.3}$$

which implies that

$$P = \lambda\left[\left(\sum_{n=1}^{N+1}\beta_n\right)D + \sum_{n=1}^{N+1}\beta_n\varepsilon_n\right], \tag{B.4}$$

where

$$\lambda = \left(\sum_{k=1}^{N+1}\gamma_k\right)^{-1}. \tag{B.5}$$

Moreover, following Kyle (1989), it is straightforward to show that

$$X_n = \frac{\mathbb{E}\left[D - P \middle| s_n, P\right]}{\lambda_{-n} + \delta_n \mathrm{Var}\left(D - P \middle| s_n, P\right)}, \tag{B.6}$$

where $\lambda_{-n}$ is the slope of the residual supply curve for trader $n$, given as

$$\lambda_{-n} = \left( \sum_{\substack{k=1 \\ k \neq n}}^{N+1} \gamma_k \right)^{-1}. \tag{B.7}$$

Let the demand coefficients of the risk-neutral traders be $\beta_{RN}$ and $\gamma_{RN}$, and the demand coefficients of the risk seeker be $\beta_{RS}$ and $\gamma_{RS}$. Moreover, let

$$\bar{\tau}_D = (N+1)\tau + \tau_D. \tag{B.8}$$

Deriving the conditional moments in (B.6) and matching coefficients with (B.2), gives, after some algebra, the following equations:

$$\beta_{RS} = N\beta_{RN} \frac{\tau\gamma_{RN}}{\beta_{RN}\bar{\tau}_D + N\gamma_{RN}(\tau + \delta\beta_{RN})} \tag{B.9}$$

and

$$\gamma_{RS} = N\gamma_{RN} \frac{\beta_{RN}\bar{\tau}_D - N\tau\gamma_{RN}}{\beta_{RN}\bar{\tau}_D + N\gamma_{RN}(\tau + \delta\beta_{RN})}, \tag{B.10}$$

where the coefficients of the risk-neutral traders are given as the solution to the system

$$\begin{aligned} \beta_{RN}^3 \left[\delta\gamma_{RN}N + \bar{\tau}_D\right]^3 (N-1)\left(\bar{\tau}_D - \tau\right) + \beta_{RN}^2\gamma_{RN}\left[\delta\gamma_{RN}N + \bar{\tau}_D\right]^2 (N-1)N\tau(3\bar{\tau}_D - \tau) \\ + \beta_{RN}\gamma_{RN}^2\left[\delta\gamma_{RN}N + \bar{\tau}_D\right]\left[\delta\gamma_{RN}N(N-1) + \left(3N^2 - 1\right)\bar{\tau}_D\right] N\tau^2 \\ + \gamma_{RN}^3\left[-\delta\gamma_{RN}N + (N^2-1)\bar{\tau}_D\right] N^2\tau^3 = 0, \end{aligned} \tag{B.11a}$$

and

$$\beta_{RN}^{3}\left(\delta\gamma_{RN}N+\bar{\tau}_{D}\right)^{2}\left[\delta\gamma_{RN}N(N-2)+2\left(N-1\right)\bar{\tau}_{D}\right]\left(N-1\right)\left(\bar{\tau}_{D}-\tau\right)$$
$$-\beta_{RN}^{2}\gamma_{RN}\left(\delta\gamma_{RN}N+\bar{\tau}_{D}\right)\left\{\delta^{2}\gamma_{RN}^{2}N^{2}(N-1)+\delta\gamma_{RN}N\left[(2N+3)\bar{\tau}_{D}-2\tau\right]+2\bar{\tau}_{D}\left(2\bar{\tau}_{D}-\tau\right)\right\}(N-1)N\tau$$
$$-\beta_{RN}\gamma_{RN}^{2}\left(\delta\gamma_{RN}N+\bar{\tau}_{D}\right)\left[\delta\gamma_{RN}N(N-1)^{2}+2\left(N^{2}+N-1\right)\bar{\tau}_{D}\right]N^{2}\tau^{2}$$
$$+\delta\gamma_{RN}^{4}N^{5}\tau^{3}=0. \quad \text{(B.11b)}$$

The second-order condition of each trader is satisfied if and only if

$$2\lambda_{-n}+\delta_{n}\mathrm{Var}\left(D-P\middle|s_{n},P\right). \quad \text{(B.12)}$$

For the risk-neutral traders this is equivalent to

$$(N-1)\gamma_{RN}+\gamma_{RS}>0, \quad \text{(B.13)}$$

while for the risk seeker it is equivalent to

$$\frac{2}{N\gamma_{RN}}+\frac{\delta}{\bar{\tau}_{D}}>0. \quad \text{(B.14)}$$

Figure 4 shows the only equilibrium for which the second-order conditions of all traders are satisfied, under the values $\tau_D = 1$, $\tau = 1$, $2 \leq N \leq 30$, with $\delta = -1$ and $\delta = -2$.

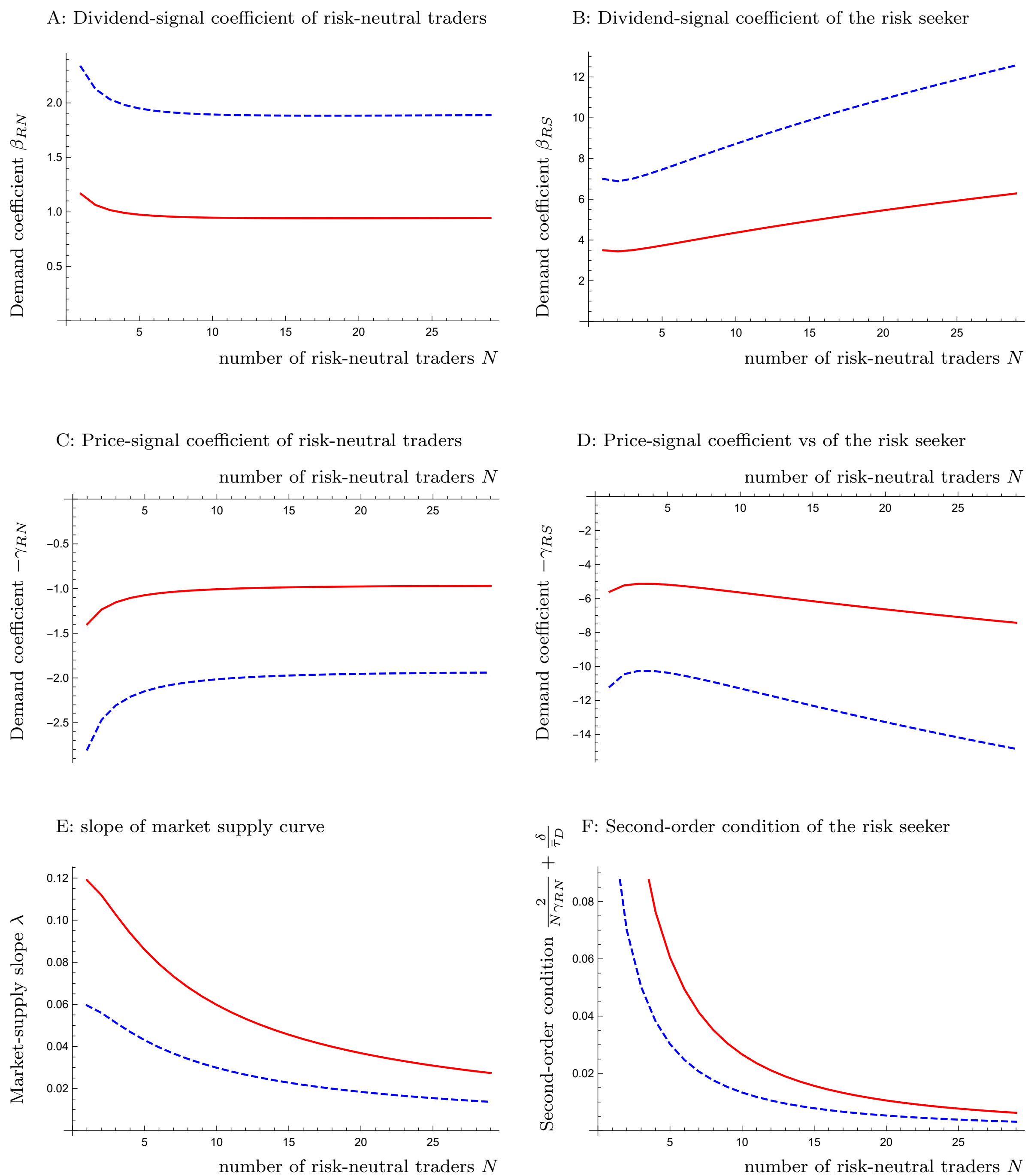


Figure 4: Solution of equilibrium with one risk seeker and $N$ risk-neutral traders and observable prices. The dashed blue curves show demand coefficients, the slope of the market supply curve ($\lambda$), and the risk seeker's second-order condition with risk aversion set to $\delta = -1$ , while the solid red curves show the same quantities with risk aversion set to $\delta = -2$. The precision of the dividend is $\tau_D = 1$, and the precision of the signal noise is $\tau = 1$ for all traders.

# C Detailed proofs of the comparative statics with respect to risk seeking

An illustration of the results of Corollary 12 is in Figure 5.

**Proof of Corollary 12.** The proof of uses the two-dimensional version of the Implicit Function Theorem. We have

$$\frac{d\rho}{d\phi} = -\frac{\begin{vmatrix} \frac{\partial}{\partial\phi}G & \frac{\partial}{\partial\int\beta}G \\ \frac{\partial}{\partial\phi}J & \frac{\partial}{\partial\int\beta}J \end{vmatrix}}{\begin{vmatrix} \frac{\partial}{\partial\rho}G & \frac{\partial}{\partial\int\beta}G \\ \frac{\partial}{\partial\rho}J & \frac{\partial}{\partial\int\beta}J \end{vmatrix}}; \quad \frac{d\rho}{d\chi} = -\frac{\begin{vmatrix} \frac{\partial}{\partial\chi}G & \frac{\partial}{\partial\int\beta}G \\ \frac{\partial}{\partial\chi}J & \frac{\partial}{\partial\int\beta}J \end{vmatrix}}{\begin{vmatrix} \frac{\partial}{\partial\rho}G & \frac{\partial}{\partial\int\beta}G \\ \frac{\partial}{\partial\rho}J & \frac{\partial}{\partial\int\beta}J \end{vmatrix}} \tag{C.1a}$$

and

$$\frac{d\int\beta}{d\phi} = -\frac{\begin{vmatrix} \frac{\partial}{\partial\phi}G & \frac{\partial}{\partial\rho}G \\ \frac{\partial}{\partial\phi}J & \frac{\partial}{\partial\rho}J \end{vmatrix}}{\begin{vmatrix} \frac{\partial}{\partial\int\beta}G & \frac{\partial}{\partial\rho}G \\ \frac{\partial}{\partial\int\beta}J & \frac{\partial}{\partial\rho}J \end{vmatrix}}; \quad \frac{d\int\beta}{d\chi} = -\frac{\begin{vmatrix} \frac{\partial}{\partial\chi}G & \frac{\partial}{\partial\rho}G \\ \frac{\partial}{\partial\chi}J & \frac{\partial}{\partial\rho}J \end{vmatrix}}{\begin{vmatrix} \frac{\partial}{\partial\int\beta}G & \frac{\partial}{\partial\rho}G \\ \frac{\partial}{\partial\int\beta}J & \frac{\partial}{\partial\rho}J \end{vmatrix}}, \tag{C.1b}$$

where, by Lemma II.2,

$$\frac{\partial}{\partial\rho}G = 4\rho - 3\left(\phi - \frac{1}{2}\right)\chi, \tag{C.2a}$$

$$\frac{\partial}{\partial\int\beta}G = 0, \tag{C.2b}$$

$$\frac{\partial}{\partial\phi}G = -\left[3\chi\rho + (1-2\phi)\chi^2\right] = -\chi\left[3\rho + (1-2\phi)\chi\right], \tag{C.2c}$$

$$\frac{\partial}{\partial\chi}G = 3\left(\frac{1}{2} - \phi\right)\rho + 2\phi(\phi-1)\chi = -\frac{1}{\chi}\left[3\left(\phi - \frac{1}{2}\right)\chi\rho + 2\phi(1-\phi)\chi^2\right], \tag{C.2d}$$

and

$$\frac{\partial}{\partial\rho}J = \left(\int_0^1 \beta(s)ds\right)^2 + 2\frac{\psi}{\left[2\rho-\phi\chi\right]^2\left[2\rho+(1-\phi)\chi\right]^2}\left[4\rho+(1-2\phi)\chi\right], \tag{C.3a}$$

$$\frac{\partial}{\partial\int\beta}J = 2\rho\int_0^1 \beta(s)ds + \tau_D, \tag{C.3b}$$

$$\frac{\partial}{\partial\phi}J = -\chi\frac{\psi}{\left[2\rho-\phi\chi\right]^2\left[2\rho+(1-\phi)\chi\right]^2}\left[4\rho+(1-2\phi)\chi\right], \tag{C.3c}$$

$$\frac{\partial}{\partial\chi}J = 2\psi\frac{\rho(1-2\phi)+\phi(\phi-1)\chi}{\left[2\rho-\phi\chi\right]^2\left[2\rho+(1-\phi)\chi\right]^2}. \tag{C.3d}$$

I note that in equilibrium

$$3\rho+(1-2\phi)\chi > 0, \tag{C.4a}$$

$$\frac{\partial}{\partial\rho}G = 4\rho - 3\left(\phi-\frac{1}{2}\right)\chi > 0, \tag{C.4b}$$

$$3\left(\phi-\frac{1}{2}\right)\chi\rho + 2\phi(1-\phi)\chi^2 = 2\rho^2+\phi(1-\phi)\chi^2 > 0, \tag{C.4c}$$

and

$$\begin{aligned}(3-2\phi)\,\chi\phi + 2\rho(2\phi-1) &= 3\chi\phi - 2\chi\phi^2 + 4\rho\left(\phi-\frac{1}{2}\right)\\ &= 3\chi\phi - 2\chi\phi^2 + \frac{8\rho^2}{3\chi} - \frac{4}{3}\phi(1-\phi)\chi = \frac{1}{3}\left[\chi\phi\,(5-2\phi)+\frac{8\rho^2}{\chi}\right] > 0.\end{aligned} \tag{C.4d}$$

The inequality in (C.4a) holds in equilibrium, because at the root for $\rho$ we have $3\rho+(1-2\phi)\chi = 2\rho-\phi\chi+\rho+(1-2\phi)\chi > 2\rho-\phi\chi > 0$ by (118). The inequality in (C.4b) holds in equilibrium because it is the slope of a quadratic polynomial with positive leading coefficient at its largest root $\rho$, which is a positive quantity. The equality in (C.4c) holds because of (113a), and the inequality in (C.4c) follows because $\rho,\chi > 0$ and $0 < \phi < 1$. The first and third equalities in (C.4d) follow by algebra, the second equality in (C.4d) holds because of (113a), and the inequality in (C.4d) holds in equilibrium because $\chi > 0$ and $0 < \phi < 1$.

**Proof of (i).** From the above we get that

$$\frac{d\rho}{d\phi} = -\frac{\frac{\partial}{\partial\phi}G}{\frac{\partial}{\partial\rho}G} = \frac{3\rho + (1-2\phi)\chi}{\frac{\partial}{\partial\rho}G}\chi. \tag{C.5}$$

The numerator of (C.5) is positive by (C.4a), and the denominator is positive by (C.4b). This proves that $d\rho/d\phi > 0$ in equilibrium.

We also get that

$$\frac{d\rho}{d\chi} = -\frac{\frac{\partial}{\partial\chi}G}{\frac{\partial}{\partial\rho}G} = \frac{1}{\chi}\frac{3\left(\phi - \frac{1}{2}\right)\chi\rho + 2\phi(1-\phi)\chi^2}{\frac{\partial}{\partial\rho}G}. \tag{C.6}$$

The numerator of (C.6) is positive by (C.4c), and the denominator is positive by (C.4b). This proves that $d\rho/d\chi > 0$ in equilibrium.

**Proof of (ii).** We have that

$$\frac{d\int\beta}{d\phi} = \frac{\frac{\partial}{\partial\phi}G\frac{\partial}{\partial\rho}J - \frac{\partial}{\partial\rho}G\frac{\partial}{\partial\phi}J}{\frac{\partial}{\partial\rho}G\frac{\partial}{\partial\int\beta}J}$$
$$= -\chi\frac{[3\rho + (1-2\phi)\chi]\left(\int_0^1\beta(s)ds\right)^2 + 2\psi\frac{\left[2\rho + (\frac{1}{2}-\phi)\chi\right]^2}{[2\rho-\phi\chi]^2[2\rho+(1-\phi)\chi]^2}}{\frac{\partial}{\partial\rho}G\left[2\rho\int_0^1\beta(s)ds + \tau_D\right]}. \tag{C.7}$$

Similarly to above, the numerator of (C.7) is positive by (C.4a), and the denominator of (C.7) is positive by (C.4b). It follows that $d\int_0^1\beta(s)ds/d\phi < 0$.

In addition,

$$\frac{d\int\beta}{d\chi} = \frac{\frac{\partial}{\partial\chi}G\frac{\partial}{\partial\rho}J - \frac{\partial}{\partial\rho}G\frac{\partial}{\partial\chi}J}{\frac{\partial}{\partial\rho}G\frac{\partial}{\partial\int\beta}J}$$

$$= \frac{-\frac{1}{\chi}\left[3\left(\phi-\frac{1}{2}\right)\chi\rho + 2\phi(1-\phi)\chi^2\right]\left(\int_0^1\beta(s)ds\right)^2 - \psi\frac{\chi^2\phi+(2\rho-\phi\chi)[(3-2\phi)\chi\phi+2\rho(2\phi-1)]}{[2\rho-\phi\chi]^2[2\rho+(1-\phi)\chi]^2}}{\frac{\partial}{\partial\rho}G\left[2\rho\int_0^1\beta(s)ds+\tau_D\right]}. \quad \text{(C.8)}$$

The first term in the numerator is negative by (C.4c), and the second term in the numerator is negative by (C.4d). The denominator is positive by (C.4b). This proves that $d\int_0^1\beta(s)ds/d\chi < 0$.

**Proof of (iii).** By Lemma 8 and Theorem 9, the trading intensity of trader $a$ is

$$\beta(a) = \frac{\frac{\psi(a)}{[\delta(a)+2\rho]^2}}{\int_0^1\frac{\psi(s)}{[\delta(s)+2\rho]^3}ds}\frac{\int_0^1\beta(s)ds}{\rho}. \quad \text{(C.9)}$$

Let

$$B(a) = \frac{\frac{\psi(a)}{[\delta(a)+2\rho]^2}}{\int_0^1\frac{\psi(s)}{[\delta(s)+2\rho]^3}ds}. \quad \text{(C.10)}$$

By (i) and (ii) of this Corollary, it suffices to prove that $B(a)$ is a decreasing function of $\phi$ for every $a$. By specification (24) we obtain

$$B(a) = \frac{2\left[2\rho-\phi\chi\right]^2\left[2\rho+(1-\phi)\chi\right]^2}{\left[4\rho+(1-2\phi)\chi\right]\left[2\rho+(a-\phi)\chi\right]^2}, \quad \text{(C.11)}$$

which implies that

$$\frac{d}{d\phi}B(a) = -\frac{4\chi\left[2\rho-\phi\chi\right]\left[2\rho+(1-\phi)\chi\right]}{\left[4\rho+(1-2\phi)\chi\right]^2\left[2\rho+(a-\phi)\chi\right]^3}\cdot$$
$$\left\{(2\rho-\phi\chi)^3 + a\chi\left[12\rho^2+6(1-2\phi)\chi\rho-3\phi(1-\phi)\chi^2+\chi^2\right]\right\} \quad \text{(C.12)}$$

By Proposition 7, we have $2\rho + (a-\phi)\chi$ for every $a \in [0,1]$, and by (C.4a), we have that

$$4\rho + (1-2\phi)\chi > 3\rho + (1-2\phi)\chi > 0. \tag{C.13}$$

Because $\chi > 0$ by assumption, it suffices to prove that

$$12\rho^2 + 6(1-2\phi)\chi\rho - 3\phi(1-\phi)\chi^2 + \chi^2 > 0. \tag{C.14}$$

Substituting the equilibrium condition (113a) into the above we obtain

$$12\rho^2 + 6(1-2\phi)\chi\rho - 3\phi(1-\phi)\chi^2 + \chi^2 = \frac{3}{2}\left[4\rho + (1-2\phi)\chi\right]\rho + \chi^2 > 0, \tag{C.15}$$

where the inequality follows from (C.13).

By (C.9), we can also write the trading intensity of trader $a$ as

$$\beta(a) = \tilde{B}(a)\int_0^1 \beta(s)ds, \tag{C.16}$$

where

$$\tilde{B}(a) = \frac{\frac{\psi(a)}{[\delta(a)+2\rho]^2}}{\rho\int_0^1 \frac{\psi(s)}{[\delta(s)+2\rho]^3}ds}. \tag{C.17}$$

By specification (24) we obtain

$$\tilde{B}(a) = \frac{2\left[2\rho - \phi\chi\right]^2\left[2\rho + (1-\phi)\chi\right]^2}{\rho\left[4\rho + (1-2\phi)\chi\right]\left[2\rho + (a-\phi)\chi\right]^2} = \frac{2\left[2\frac{\rho}{\chi} - \phi\right]^2\left[2\frac{\rho}{\chi} + (1-\phi)\right]^2}{\frac{\rho}{\chi}\left[4\frac{\rho}{\chi} + (1-2\phi)\right]\left[2\frac{\rho}{\chi} + (a-\phi)\right]^2}. \tag{C.18}$$

In addition, from the equilibrium condition (113a) we get that

$$2\left(\frac{\rho}{\chi}\right)^2 - 3\left(\phi - \frac{1}{2}\right)\frac{\rho}{\chi} - \phi(1-\phi) = 0. \tag{C.19}$$

This proves that the solution for $\rho/\chi$ does not depend on $\chi$, because (C.19) is a quadratic polynomial in $\rho/\chi$ where the coefficients do not depend on $\chi$. It follows that $\tilde{B}(a)$ does not depend on $\chi$ either. By (ii) of this Corollary, it follows that $\beta(a)$ decreases in $\chi$ for all $a$.

**Proof of (iv).** Liquidity $\lambda^{-1}$ decreases in $\phi$ and $\chi$ by Equation (18b) and (i) and (ii) of this Corollary. $\mathcal{V}$ decreases in $\phi$ and $\chi$ due to (22a) and Corollary 12.

Next, by (21), the signal-to-noise ratio is

$$\mathcal{Q} = \frac{\mathrm{Var}\left(D\int_0^1 \beta(a)da\right)}{\mathrm{Var}\left(\int_0^1 \beta(a)\sqrt{\tau(a)^{-1}}dB_a\right)} = \frac{\rho\int_0^1 \beta(a)da}{\tau_D}, \tag{C.20}$$

where the second equality follows by (91). By (C.5) and (C.7), Equation (C.20) gives

$$\begin{aligned}
\tau_D\frac{d}{d\phi}\mathcal{Q} &= \rho\frac{d}{d\phi}\left(\int_0^1 \beta(s)ds\right) + \left(\int_0^1 \beta(s)ds\right)\frac{d}{d\phi}\rho \\
&= \chi\frac{[3\rho + (1-2\phi)\chi]\left[\rho\left(\int_0^1 \beta(s)ds\right)^2 + \tau_D\int_0^1 \beta(s)ds\right] - 2\psi\frac{\left[2\rho+(\frac{1}{2}-\phi)\chi\right]^2}{[2\rho-\phi\chi]^2[2\rho+(1-\phi)\chi]^2}}{\frac{\partial}{\partial\rho}G\left[2\rho\int_0^1 \beta(s)ds + \tau_D\right]} \\
&= \chi\psi\frac{[3\rho + (1-2\phi)\chi]\,[2\rho-\phi\chi]\,[2\rho+(1-\phi)\chi] - 2\left[2\rho+(\frac{1}{2}-\phi)\chi\right]^2}{\frac{\partial}{\partial\rho}G\left[2\rho\int_0^1 \beta(s)ds + \tau_D\right][2\rho-\phi\chi]^2\,[2\rho+(1-\phi)\chi]^2}.
\end{aligned} \tag{C.21}$$

The second equality is due to the equilibrium condition (113b). By (115) we obtain that

$$\rho\left[2\rho + \left(\frac{1}{2}-\phi\right)\chi\right] = [2\rho-\phi\chi]\,[2\rho+(1-\phi)\chi]\,, \tag{C.22}$$

and substituting this into (C.25) we get

$$\tau_D \frac{d}{d\phi}\mathcal{Q} = \chi\psi \frac{-\left[3\rho + (1-2\phi)\chi\right]^2 \left[2\rho + (\frac{1}{2}-\phi)\chi\right]}{\frac{\partial}{\partial\rho}G\left[2\rho\int_0^1 \beta(s)ds + \tau_D\right]\left[2\rho - \phi\chi\right]^2\left[2\rho + (1-\phi)\chi\right]^2} < 0, \tag{C.23}$$

because

$$2\rho + \left(\frac{1}{2} - \phi\right)\chi = 2\rho - \phi\chi + \frac{1}{2}\chi > 0 \tag{C.24}$$

in equilibrium. It follows that $\mathcal{Q}$ is a decreasing function of $\phi$.

A similar argument shows that

$$\begin{aligned}
\tau_D \frac{d}{d\chi}\mathcal{Q} &= \rho\frac{d}{d\chi}\left(\int_0^1 \beta(s)ds\right) + \left(\int_0^1 \beta(s)ds\right)\frac{d}{d\chi}\rho \\
&= \frac{\psi}{\frac{\partial}{\partial\rho}G\left[2\rho\int_0^1 \beta(s)ds + \tau_D\right]\left[2\rho-\phi\chi\right]^2\left[2\rho+(1-\phi)\chi\right]^2} \\
&\quad \left\{\left[3\left(\phi - \frac{1}{2}\right)\rho + 2\phi(1-\phi)\chi\right]\left[2\rho - \phi\chi\right]\left[2\rho + (1-\phi)\chi\right]\right. \\
&\quad \left. - \rho\left\{\chi^2\phi + (2\rho - \phi\chi)\left[(3-2\phi)\chi\phi + 2\rho(2\phi-1)\right]\right\}\right\} \\
&= \frac{\psi}{\frac{\partial}{\partial\rho}G\left[2\rho\int_0^1 \beta(s)ds + \tau_D\right]\left[2\rho-\phi\chi\right]^2\left[2\rho+(1-\phi)\chi\right]^2} \\
&\quad \rho\left\{\left[3\left(\phi - \frac{1}{2}\right)\rho + 2\phi(1-\phi)\chi\right]\left[2\rho + \left(\frac{1}{2} - \phi\right)\chi\right]\right. \\
&\quad \left. - \left\{\chi^2\phi + (2\rho - \phi\chi)\left[(3-2\phi)\chi\phi + 2\rho(2\phi-1)\right]\right\}\right\} \\
&= -\frac{\psi\rho(1-\phi)\left[2\left(3\rho - \phi\chi\right) + 4\phi\chi\rho\right]}{4\frac{\partial}{\partial\rho}G\left[2\rho\int_0^1 \beta(s)ds + \tau_D\right]\left[2\rho-\phi\chi\right]^2\left[2\rho+(1-\phi)\chi\right]^2},
\end{aligned} \tag{C.25}$$

where the first equality follows from (C.6), (C.8) and equilibrium condition (113b), the second equality follows from (C.22), and the third equality follows from (113a). The numerator in

(C.25) is positive because $\phi, \chi, \rho > 0$, and $3\rho - \phi\chi > 2\rho - \phi\chi > 0$ in equilibrium. It now follows that $\mathcal{Q}$ is a decreasing function of $\chi$. ■

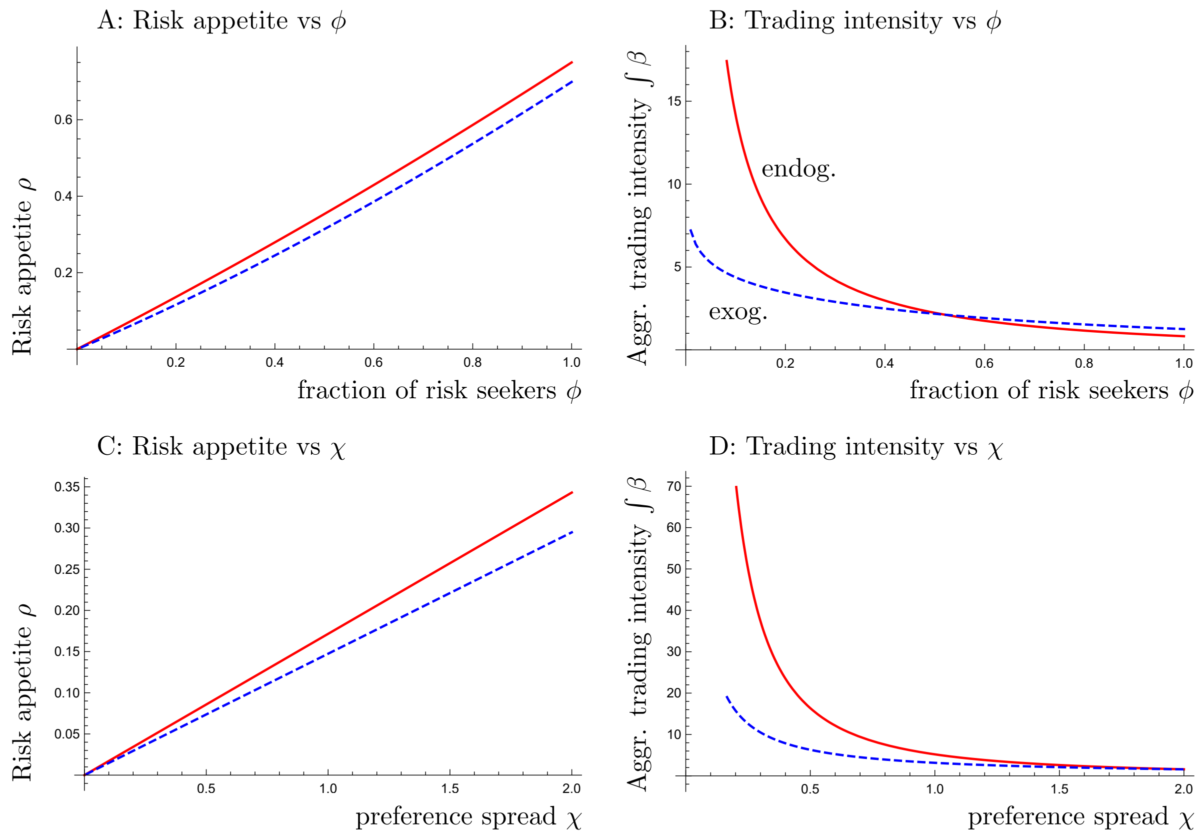


Figure 5: An illustration of the results in Corollary 12. Risk appetite $\rho$ on the left (Panels A and C) and aggregate trading intensity $\int \beta$ on the right (Panels B and D) as a function of the fraction of risk seekers $\phi$ and the preference spread $\chi$. The dashed blue curves show the equilibrium with exogenous homogeneous precisions set to $\tau(a) = 1$ for all traders $a$, while the solid red curves show the equilibrium with endogenous precisions. The precision of the dividend is $\tau_D = 1$ and the inverse of the marginal cost of information is $\psi(a) = 1$ for all $a$. In Panels A and B the preference spread is $\chi = 1$ and in Panels C and D the fraction of risk seekers is $\phi = 1/4$.

# D Detailed proofs for the heterogeneous market with price observations and imperfect competition (Section 3.5)

**Theorem 14** *Given precision functions $\tau$ and $\pi$, if an equilibrium exists, then it must satisfy*

$$\beta(a) = \frac{\tau(a)}{2\rho + \mu\delta(a)}, \tag{D.1a}$$

$$\gamma(a) = \frac{\pi(a)}{2\rho + \mu\delta(a)}(\mu - 1), \tag{D.1b}$$

*and*

$$\frac{1}{\lambda} = \int_0^1 \beta(s)ds + \frac{\tau_D}{\rho}, \tag{D.1c}$$

*where, with $\rho_* = \rho/\mu$, $\rho_*$ and $\mu$ are the solution to the system*

$$\rho_* = \left\{\int_0^1 \frac{\tau(s)}{[2\rho_* + \delta(s)]^2}ds + (\mu-1)^2 \int_0^1 \frac{\pi(s)}{[2\rho_* + \delta(s)]^2}ds\right\}^{-1} \cdot \left[\int_0^1 \frac{\tau(s)}{2\rho_* + \delta(s)}ds + (\mu-1)^2 \int_0^1 \frac{\pi(s)}{2\rho_* + \delta(s)}ds\right], \tag{D.1d}$$

*and*

$$\mu = 1 + \left[\int_0^1 \frac{\pi(s)}{2\rho_* + \delta(s)}ds\right]^{-1} \left[\frac{\tau_D}{\rho_*} + \int_0^1 \frac{\tau(s)}{2\rho_* + \delta(s)}ds\right]. \tag{D.1e}$$

*under the restriction that $\rho_*$ and $\lambda$ have the same sign. Moreover, the second-order condition of trader $a$ is satisfied if and only if $2\rho_* + \delta(a) > 0$.*

**Proof.** By market clearing we have

$$\lambda = \left[\int_0^1 \gamma(s)ds\right]^{-1}. \tag{D.2}$$

Following the methodology of Kyle (1989), we can write

$$\lambda_{-a} = \frac{\lambda}{1 - \lambda\gamma(a)da} \tag{D.3}$$

and

$$P_{-a} = \lambda_{-a}\Bigg(D\int_0^1 \mathbb{I}_{\{s\neq a\}}\beta(s)ds + \int_0^1 \mathbb{I}_{\{s\neq a\}}\beta(s)\sqrt{\tau(s)^{-1}}dB_s^z + \int_0^1 \mathbb{I}_{\{s\neq a\}}\gamma(s)\sqrt{\pi(s)^{-1}}dB_s^\zeta\Bigg). \tag{D.4}$$

Conditional on the signals $dz_a$ and $d\zeta_a$, the utility that trader $a$ gets from his profit $\pi_a = dX_a(D-P)$ is

$$\begin{aligned} du\left(\pi_a; dz_a\right) &= \mathbb{E}\left[\pi_a\middle| dz_a, d\zeta_a\right] - \frac{1}{2}\delta(a)\mathrm{Var}\left(\pi_a\middle| dz_a, d\zeta_a\right) \\ &= dX_a\left(\mathbb{E}\left[D - P_{-a}\middle| dz_a, d\zeta_a\right] - \lambda\, dX_a\right) - \frac{1}{2}\delta(a)\left(dX_a\right)^2\mathrm{Var}\left(D - P_{-a}\middle| dz_a, d\zeta_a\right). \end{aligned} \tag{D.5}$$

The first-order condition of maximizing utility over $dX_a$ gives

$$dX_a = \frac{\mathbb{E}\left[D - P_{-a}\middle| dz_a, d\zeta_a\right]}{2\lambda_{-a} + \delta(a)\mathrm{Var}\left(D - P_{-a}\middle| dz_a, d\zeta_a\right)}. \tag{D.6}$$

Combining this expression with (119) we get (120).

I conjecture that

$$\lim_{da\to 0}\lambda \tag{D.7a}$$

is finite, and that for each $a$

$$\lim_{da\to 0}\gamma(a) \tag{D.7b}$$

is also finite. These conjectures imply that

$$\lim_{da \to 0} \lambda_{-a} = \lambda \tag{D.8}$$

for all $a$.

Deriving the conditional expectation of $D - P_{-a}$ we get an expression for $dX_a$ as a linear combination of $dz_a$ and $d\zeta_a$. Matching its coefficients with $\beta(a)$ and $-\gamma(a)$ and sending $da$ to zero under the conjectures in (D.7), we obtain

$$\beta(a) = \frac{1}{\lambda + \delta(a)\mathrm{Var}\left(D - P_{-a} \middle| dz_a, d\zeta_a\right)} \left[-\lambda\beta(a) + \frac{\tau(a)}{\tau_D}\left(1 - \lambda\int_0^1 \beta(s)ds\right)\right] \tag{D.9a}$$

and

$$\begin{aligned} -\gamma(a) = & \frac{1}{\lambda + \delta(a)\mathrm{Var}\left(D - P_{-a} \middle| dz_a, d\zeta_a\right)} \\ & \left[\lambda\gamma(a) + \frac{\pi(a)}{\tau_D}\lambda\int_0^1 \beta(s)ds\left(1 - \lambda\int_0^1 \beta(s)ds\right) - \pi(a)\lambda^2\int_0^1\left(\frac{\beta^2(s)}{\tau(s)} + \frac{\gamma^2(s)}{\pi(s)}\right)ds\right] \end{aligned} \tag{D.9b}$$

Solving (D.9a) for $\beta(a)$ we get (D.1a) and solving (D.9b) for $\gamma(a)$ we get (D.1b), where the auxiliary quantities $\rho$ and $\mu$ are

$$\rho = \frac{\lambda\tau_D}{1 - \lambda\int_0^1 \beta(s)ds} \tag{D.10a}$$

and

$$\mu = \lim_{da\to 0} \frac{\tau_D\mathrm{Var}\left(D - P_{-a}\middle| dz_a, d\zeta_a\right)}{1 - \lambda\int_0^1 \beta(s)ds} = 1 - \lambda\int_0^1 \beta(s)ds + \tau_D\frac{\lambda^2\int_0^1\left(\frac{\beta^2(s)}{\tau(s)} + \frac{\gamma^2(s)}{\pi(s)}\right)ds}{1 - \lambda\int_0^1 \beta(s)ds}. \tag{D.10b}$$

Equation (D.10a) implies (D.1c).

Because (D.2) must hold in equilibrium, it follows that the function $\gamma$ is finite everywhere if an equilibrium exists. Thus the conjectures in (D.7) hold as long as an equilibrium exists, which implies that (D.1a) and (D.1b) also hold as long as an equilibrium exists.

From (D.10a) and (D.2) we get

$$\rho = \frac{\tau_D}{\int_0^1 \gamma(s)ds - \int_0^1 \beta(s)ds} = \frac{\tau_D}{(\mu-1)\int_0^1 \frac{\pi(s)}{2\rho+\mu\delta(s)}ds - \int_0^1 \frac{\tau(s)}{2\rho+\mu\delta(s)}ds} \tag{D.11}$$

Rearranging this expression gives (D.1e), with $\rho_*$ defined as $\rho$ scaled by $\mu$.

Substituting (D.10a) and (D.2) into (D.10b) we get

$$\rho = \frac{\int_0^1 \beta(s)ds + (\mu-1)\int_0^1 \gamma(s)ds}{\int_0^1 \left(\frac{\beta^2(s)}{\tau(s)} + \frac{\gamma^2(s)}{\pi(s)}\right) ds}. \tag{D.12}$$

Using (D.1a) and (D.1b) to write out the integrals that appear in the above expression we obtain

$$\rho = \left\{\int_0^1 \frac{\tau(s)}{[2\rho+\mu\delta(s)]^2}ds + (\mu-1)^2\int_0^1 \frac{\pi(s)}{[2\rho+\mu\delta(s)]^2}ds\right\}^{-1} \cdot \left[\int_0^1 \frac{\tau(s)}{2\rho+\mu\delta(s)}ds + (\mu-1)^2\int_0^1 \frac{\pi(s)}{2\rho+\mu\delta(s)}ds\right], \tag{D.13}$$

and scaling (D.13) by $\mu$ we get (D.1d).

Dividing (D.10a) and (D.10b) implies that $\lambda > 0$ if and only if $\rho$ and $\mu$ have the same sign, which shows that $\lambda$ and $\rho_*$ have the same sign.

Finally, because $\mu \geq 1$ the second-order condition of trader $a$ is satisfied if and only if $2\rho_* + \delta(a) > 0$. ■

**Proof of Theorem 13.** If the ratio $\frac{\pi(s)}{\tau(s)}$ depends on $s$, then the equilibrium is described by Theorem 14.

If the ratio $\frac{\pi(s)}{\tau(s)}$ is constant and equal to $\omega$, then (D.1d) reduces to (18c) with $\rho_* = \frac{\rho}{\mu}$ in the place of $\rho$. After scaling by $\mu$ and using $\frac{\pi(s)}{\tau(s)} = \omega$ (D.1a) gives (34a), (D.1b) gives (34b), (D.1c) gives (34c), and (D.1e) gives (34d).

If prices are fully observable, then $\pi = \infty$. Letting $\pi(s) = \pi$ for all $s$ and sending $\pi \to \infty$ in (D.1e) shows that $\mu \to 1$, and that

$$\lim_{\pi \to \infty} (\mu - 1)\,\pi = \lim_{\pi \to \infty} \left[ \int_0^1 \frac{1}{2\rho_* + \delta(s)} ds \right]^{-1} \left[ \frac{\tau_D}{\rho_*} + \int_0^1 \frac{\tau(s)}{2\rho_* + \delta(s)} ds \right]. \tag{D.14}$$

I conjecture that this limit exists. Because $\mu \to 1$, (D.1d) implies that $\rho_*$ converges to the solution of

$$\rho_* = \left\{ \int_0^1 \frac{\tau(s)}{[2\rho_* + \delta(s)]^2} ds \right\}^{-1} \left[ \int_0^1 \frac{\tau(s)}{2\rho_* + \delta(s)} ds \right], \tag{D.15}$$

that is, $\rho_*$ converges to the $\rho$ of Theorem 4. Thus the limit in (D.14) exists if an equilibrium in Theorem 4 exists. The claims for $\beta$ and $\gamma$ follow immediately. ■

There is also a direct extension of Theorem 6.

**Theorem 15** *For given precisions functions $\tau$ and $\pi$, if $\delta(a)$ is non-negative for all $a$ then no equilibrium exists in which the second-order condition of all traders is strictly satisfied.*

**Proof.** The proof is by contradiction; it follows a similar method to that for Theorem 6.

Suppose that $\rho_* < 0$. Because $\tau > 0$ and $\pi > 0$, Equation (D.1d) implies that the second-order condition for a set of traders of positive measure must be violated.

Suppose that $\rho_* = 0$. If $\delta(a) = 0$ for at least some traders $a$ of non-zero measure, then the second-order condition is weakly violated for those traders. If, instead, $\delta(a) > 0$ for all

$a$, Equation (D.1d) implies that

$$0 = \int_0^1 \frac{\tau(s)}{\delta(s)} ds + (\mu - 1)^2 \int_0^1 \frac{\pi(s)}{\delta(s)} ds, \tag{D.16}$$

which is not possible because $\tau > 0$ and $\pi > 0$.

Suppose that $\rho_* > 0$. If $\delta(a) = 0$ for all $a$, then (D.1d) implies

$$\rho_* = \frac{\frac{1}{2\rho_*}}{\frac{1}{4\rho_*^2}} = 2\rho_*, \tag{D.17}$$

which is a contradiction. If $\delta(a) \geq 0$, then because $\rho_* > 0$ we get that

$$0 < \frac{\rho_*}{2\rho_* + \delta(a)} \leq 1. \tag{D.18}$$

It follows that

$$0 < \rho_* \frac{\tau(a)}{[2\rho_* + \delta(a)]^2} \leq \frac{\tau(a)}{2\rho_* + \delta(a)}. \tag{D.19a}$$

and

$$0 < \rho_* \frac{(\mu - 1)^2 \pi(a)}{[2\rho_* + \delta(a)]^2} \leq \frac{(\mu - 1)^2 \pi(a)}{2\rho_* + \delta(a)}. \tag{D.19b}$$

Integrating both sides on each inequality in (D.19a) and (D.19b) and summing the resulting inequalities we obtain, because each inequality is strict for at least some $a$ of non-zero measure, that

$$\begin{aligned} \rho_* \left\{ \int_0^1 \frac{\tau(s)}{[2\rho_* + \delta(s)]^2} ds + (\mu - 1)^2 \int_0^1 \frac{\pi(s)}{[2\rho_* + \delta(s)]^2} ds \right\} \\ < \int_0^1 \frac{\tau(s)}{2\rho_* + \delta(s)} ds + (\mu - 1) \int_0^1 \frac{\pi(s)}{2\rho_* + \delta(s)} ds \end{aligned} \tag{D.20}$$

which contradicts (D.1d). ■

# E Sufficient conditions on non-linear heterogeneity in risk preferences

**Theorem 16** *Suppose that $\delta(s)$ crosses zero from below in $(0,1)$ and it increases strictly.*

*(i) For a given precision function $\tau$, a unique financial-market equilibrium with positive $\rho$ exists if*

$$\frac{\tau'(s)}{\tau(s)} < \frac{\delta''(s)}{\delta'(s)} \tag{E.1}$$

*for all $s$. Moreover, in this equilibrium the second-order condition of all traders is strictly satisfied.*

*(ii) For a given inverse marginal-cost function $\psi$, a unique information-acquisition equilibrium with positive $\rho$ exists if*

$$\frac{1}{3}\frac{\frac{\psi''(s)}{\psi(s)} - \frac{\delta'''(s)}{\delta'(s)}}{\frac{\psi'(s)}{\psi(s)} - \frac{\delta''(s)}{\delta'(s)}} < \frac{\delta''(s)}{\delta'(s)} \tag{E.2}$$

*for all $s$. Moreover, in this equilibrium the second-order condition of all traders is strictly satisfied.*

**Proof.** The proofs of both parts of the theorem use a similar method—I thus present an expanded version for (i) and an abridged version for (ii). Suppose that $s_*$ is the root of $\delta$ in $[0,1]$. This root is unique by assumption.

**Proof of (i).** At $\rho = 0$, the left-hand side of (18c) is zero, but its right-hand side equals

$$\frac{\int_0^1 \frac{\tau(s)}{\delta(s)} ds}{\int_0^1 \frac{\tau(s)}{[\delta(s)]^2} ds} = \frac{\int_0^{\delta(1)-\delta(0)} \frac{\tau\left(\delta^{-1}(u+\delta(0))\right)}{u+\delta(0)} \left(\delta^{-1}\right)'(u+\delta(0))\, du}{\int_0^{\delta(1)-\delta(0)} \frac{\tau(\delta^{-1}(u+\delta(0)))}{[u+\delta(0)]^2} \left(\delta^{-1}\right)'(u+\delta(0))\, du} = \frac{\tau\left(\delta^{-1}(0)\right)\left(\delta^{-1}\right)'(0)}{\left(\tau\left(\delta^{-1}(0)\right)\left(\delta^{-1}\right)'(0)\right)'}$$

$$= \frac{\tau\left(\delta^{-1}\left(0\right)\right)\left(\delta^{-1}\right)'\left(0\right)}{\tau'\left(\delta^{-1}\left(0\right)\right)\left[\left(\delta^{-1}\right)'\left(0\right)\right]^{2}+\tau\left(\delta^{-1}\left(0\right)\right)\left(\delta^{-1}\right)''\left(0\right)} = \delta'\left(s_*\right)\left(\frac{\tau'\left(s_*\right)}{\tau\left(s_*\right)}-\frac{\delta''\left(s_*\right)}{\delta'\left(s_*\right)}\right)^{-1}. \quad \text{(E.3)}$$

The first equality follows by change of variables, the second by Cauchy's integral formula, the third by standard formulas due to the inverse function theorem, and the fourth because $s_*$ is the root of $\delta$. By (E.1) it follows that at $\rho = 0$ the right-hand side of (18c) is strictly below its left-hand side. Moreover, the derivative of the left-hand side of (18c) is one, while the derivative of the right-hand side of (18c) is

$$2\left[2\left(\int_0^1 \frac{\tau(s)}{\left[2\rho+\delta(s)\right]^2}ds\right)^{-2}\int_0^1 \frac{\tau(s)}{2\rho+\delta(s)}ds\int_0^1 \frac{\tau(s)}{\left[2\rho+\delta(s)\right]^3}ds-1\right] \geq 2 > 1. \quad \text{(E.4)}$$

The first inequality follows because

$$\left(\int_0^1 \frac{\tau(s)}{\left[2\rho+\delta(s)\right]^2}ds\right)^2 = \left(\int_0^1 \left(\frac{\tau(s)}{2\rho+\delta(s)}\right)^{\frac{1}{2}}\left(\frac{\tau(s)}{\left[2\rho+\delta(s)\right]^3}\right)^{\frac{1}{2}}\right)^2 ds$$
$$\leq \int_0^1 \frac{\tau(s)}{2\rho+\delta(s)}ds\int_0^1 \frac{\tau(s)}{\left[2\rho+\delta(s)\right]^3}ds \quad \text{(E.5)}$$

by the Hölder inequality (by the complex version of the Hölder inequality, (E.5) holds even if the integrands are negative). Inequality (E.4) shows that the right-hand side of (18c) increases at a higher rate in $\rho$ than its left-hand side for every $\rho > 0$, and thus there must be a crossing for some $\rho > 0$. This crossing is also unique because by (E.4) the right-hand side of (18c) increases *strictly* faster in $\rho$ than its left-hand side.

Let $\rho_0 > 0$ denote the crossing in $\rho$. Assume, in contradiction, that in equilibrium there exists some $s \in [0,1]$ such that $2\rho_0 + \delta(s) \leq 0$. Let $s_0$ denote the largest such $s$; $s_0 = \delta^{-1}\left(-2\rho_0\right)$, and it exists because $\delta$ is increasing and continuous. By change of variables, Cauchy's integral formula, the inverse function theorem, and (18c) it follows that

$$\rho_0 = \frac{\int_0^1 \frac{\tau(s)}{2\rho_0+\delta(s)} ds}{\int_0^1 \frac{\tau(s)}{[2\rho_0+\delta(s)]^2} ds} = \frac{\int_{\delta(0)}^{\delta(1)} \frac{\tau\left(\delta^{-1}(u)\right)}{2\rho_0+u} \left(\delta^{-1}\right)'(u) du}{\int_{\delta(0)}^{\delta(1)} \frac{\tau(\delta^{-1}(u))}{[2\rho_0+u]^2} \left(\delta^{-1}\right)'(u) du} = \frac{\tau\left(\delta^{-1}(-2\rho_0)\right)\left(\delta^{-1}\right)'(-2\rho_0)}{\left(\tau\left(\delta^{-1}(-2\rho_0)\right)\left(\delta^{-1}\right)'(-2\rho_0)\right)'}$$

$$= \delta'(s_0)\left(\frac{\tau'(s_0)}{\tau(s_0)} - \frac{\delta''(s_0)}{\delta'(s_0)}\right)^{-1}. \quad \text{(E.6)}$$

By condition (E.1) this is negative, which contradicts that $\rho_0 > 0$.

**Proof of (ii).** By Descartes' rule of signs, it suffices for (28a) to have a positive solution for (28b) to have a positive solution. We may thus focus on conditions for (28a) to have a positive solution.

At $\rho = 0$, the left-hand side of (28a) is zero, but its right-hand side equals

$$\frac{\int_0^1 \frac{\psi(s)}{[\delta(s)]^2} ds}{\int_0^1 \frac{\psi(s)}{[\delta(s)]^3} ds} = \frac{\int_0^{\delta(1)-\delta(0)} \frac{\psi\left(\delta^{-1}(u+\delta(0))\right)}{[u+\delta(0)]^2} \left(\delta^{-1}\right)'(u+\delta(0))\, du}{\int_0^{\delta(1)-\delta(0)} \frac{\psi(\delta^{-1}(u+\delta(0)))}{[u+\delta(0)]^3} \left(\delta^{-1}\right)'(u+\delta(0))\, du} = \frac{\left(\psi\left(\delta^{-1}(0)\right)\left(\delta^{-1}\right)'(0)\right)'}{\left(\psi\left(\delta^{-1}(0)\right)\left(\delta^{-1}\right)'(0)\right)''}$$

$$= \delta'(s_*)\left[\frac{\frac{\psi''(s_*)}{\psi(s_*)} - \frac{\delta'''(s_*)}{\delta'(s_*)}}{\frac{\psi'(s_*)}{\psi(s_*)} - \frac{\delta''(s_*)}{\delta'(s_*)}} - 3\frac{\delta''(s_*)}{\delta'(s_*)}\right]^{-1} < 0, \quad \text{(E.7)}$$

and thus at $\rho = 0$ the right-hand side of (28a) is strictly below its left-hand side. In addition, the derivative of the left-hand side of (28a) is one, while the derivative of the right-hand side of (28a) is

$$2\left[3\left(\int_0^1 \frac{\psi(s)}{[2\rho+\delta(s)]^3} ds\right)^{-2} \int_0^1 \frac{\psi(s)}{[2\rho+\delta(s)]^2} ds \int_0^1 \frac{\psi(s)}{[2\rho+\delta(s)]^4} ds - 2\right] \geq 2 > 1. \quad \text{(E.8)}$$

because, by the Hölder inequality,

$$\left(\int_0^1 \frac{\psi(s)}{[2\rho+\delta(s)]^3}ds\right)^2 = \left(\int_0^1 \left(\frac{\psi(s)}{[2\rho+\delta(s)]^2}\right)^{\frac{1}{2}} \left(\frac{\psi(s)}{[2\rho+\delta(s)]^4}\right)^{\frac{1}{2}}\right)^2 ds$$
$$\leq \int_0^1 \frac{\psi(s)}{[2\rho+\delta(s)]^2}ds \int_0^1 \frac{\psi(s)}{[2\rho+\delta(s)]^4}ds. \quad \text{(E.9)}$$

This shows that there must be a unique crossing for some $\rho > 0$. Let $\rho_\dagger > 0$ denote this crossing in $\rho$ and $s_\dagger = \delta^{-1}(-2\rho_\dagger) \in [0,1]$. A similar argument as above shows that we have a contradiction, because by (E.2)

$$\rho_\dagger = \delta'(s_\dagger)\left[\frac{\frac{\psi''(s_\dagger)}{\psi(s_\dagger)} - \frac{\delta'''(s_\dagger)}{\delta'(s_\dagger)}}{\frac{\psi'(s_\dagger)}{\psi(s_\dagger)} - \frac{\delta''(s_\dagger)}{\delta'(s_\dagger)}} - 3\frac{\delta''(s_\dagger)}{\delta'(s_\dagger)}\right]^{-1} < 0. \quad \text{(E.10)}$$

■

## F A large market with noise traders

Here I introduce pure noise traders of measure $\nu$. For $a$ in $[1, 1+\nu)$, let the demand of pure noise trader $a$ be

$$dX_a = \Theta \, da + \tau_\xi^{-1/2} dB_a^\xi, \tag{F.1}$$

where $\Theta$ is $\mathcal{N}\left(0, \tau_\Theta^{-1}\right)$, and $B^\xi$ is a Brownian Motion in $[1, 1+\nu]$, independent of $\Theta$, $D$, and $B$. I use $\theta$ to denote the aggregate demand coming from noise traders. By integrating it follows that

$$\theta = \int_1^{1+\nu} dX_a = \nu\Theta + \tau_\xi^{-1/2} \int_1^{1+\nu} dB_a^\xi, \tag{F.2}$$

and thus $\theta$ is a Normal random variable with mean zero and variance

$$\tau_\theta^{-1} = \nu^2 \tau_\Theta^{-1} + \nu \tau_\xi^{-1}. \tag{F.3}$$

If the precision $\tau_\xi$ is infinite for every noise trader then there is only systematic noise, while if the precision $\tau_\Theta$ is infinite there is only idiosyncratic noise. The case of no noise traders corresponds to $\nu = 0$. It is straightforward to extend this model to heterogeneous precisions of idiosyncratic noise, but given that the only thing that matters is the aggregate variance of stochastic supply, the model above is without loss of generality.[23]

The derivation of the economy with noise traders is very similar to that without them. The demand conjecture for each informed trader $a$ in $[0, 1)$ remains as in (13), the price conjecture, which now includes the noise traders, becomes

$$P = \lambda \int_0^{1+\nu} dX_a, \tag{F.4}$$

[23] For example, if $dX_a = \Theta \, da + \sqrt{\tau_\xi(a)^{-1}} \, dB_a^\xi$, for a trader-specific amount $\tau_\xi(a)$, then $\tau_\theta^{-1} = \nu^2 \tau_\Theta^{-1} + \int_1^{1+\nu} \tau_\xi(a)^{-1} da$.

and the market maker's pricing rule becomes

$$P = \mathbb{E}\left[D \Big| \int_0^{1+\nu} dX_a\right]. \tag{F.5}$$

The following theorem summarizes the equilibrium in the financial market.

**Corollary F.1** *With homogeneous risk aversion and pure noise traders* (18a) *and* (18b) *continue to hold. Moreover, there exists a unique equilibrium with information acquisition, in which*

*(i) the adjusted market impact $\rho$ decreases in $\nu$, and*

*(ii) the aggregate trading intensity $\int \beta$ increases in $\nu$.*

*In this equilibrium, liquidity decreases when information becomes cheaper, as long as the marginal cost of information is sufficiently low. The range of information costs for which this happens becomes larger with higher risk aversion.*

## F.1 Detailed proofs

**Theorem F.2** *Given a precision function $\tau$ for the informed traders, Theorem 4 continues to hold with pure noise traders, with the only difference that Equation* (18c) *becomes*

$$\rho = \left[\int_0^1 \frac{\tau(s)}{[2\rho + \delta(s)]^2} ds + \frac{1}{\tau_\theta}\right]^{-1} \int_0^1 \frac{\tau(s)}{2\rho + \delta(s)} ds, \tag{F.6}$$

*and that the price is as in* (F.4).

**Proof.** Equation (75) remains the same. The price function without the impact of trader $a$ is

$$P_{-a} = \lambda \int_0^1 \mathbb{I}_{\{s \neq a\}} dX_s + \lambda\theta$$

$$= \left( \lambda \int_0^1 \mathbb{I}_{\{s \neq a\}} \beta(s) ds \right) D + \lambda \int_0^1 \mathbb{I}_{\{s \neq a\}} \beta(s) \sqrt{\tau(s)^{-1}} dB_s + \lambda\theta. \quad \text{(F.7)}$$

which implies that (78) still holds by independence of $\theta$ from $D$ and $B$, and that

$$\text{Var}\left(D - P_{-a}\right) = \tau_D^{-1} \left(1 - \lambda \int_0^1 \beta(s) ds\right)^2 + \lambda^2 \int_0^1 \beta^2(s)\tau(s)^{-1} ds + \lambda^2 \tau_\theta^{-1} + O(ds). \quad \text{(F.8)}$$

In Equation (80), the only term that changes is $\text{Var}\left(D - P_{-a}\right)$, which is now given by (F.8). Sending $ds$ and $da$ to zero thus yields

$$\beta(a) = \frac{\tau(a)}{2\rho + \delta(a) \left[1 - \lambda \int_0^1 \beta(s) ds + \tau_D \lambda^2 \frac{\int_0^1 \beta^2(s)\tau(s)^{-1} ds + \tau_\theta^{-1}}{1 - \lambda \int_0^1 \beta(s) ds}\right]}, \quad \text{(F.9)}$$

where the adjusted-market impact parameter $\rho$ remains as in (82), and thus (18b) continues to hold. From (F.5) we now obtain

$$\lambda = \frac{\text{Cov}\left(D, \int_0^{1+\nu} dX_s\right)}{\text{Var}\left(\int_0^{1+\nu} dX_s\right)} = \frac{\int_0^1 \beta(s) ds}{\left(\int_0^1 \beta(s) ds\right)^2 + \tau_D \left(\int_0^1 \frac{\beta^2(s)}{\tau(s)} ds + \frac{1}{\tau_\theta}\right)}, \quad \text{(F.10)}$$

from which it follows that

$$1 - \lambda \int_0^1 \beta(s) ds = \frac{\tau_D \left(\int_0^1 \frac{\beta^2(s)}{\tau(s)} ds + \frac{1}{\tau_\theta}\right)}{\left(\int_0^1 \beta(s) ds\right)^2 + \tau_D \left(\int_0^1 \frac{\beta^2(s)}{\tau(s)} ds + \frac{1}{\tau_\theta}\right)}. \quad \text{(F.11)}$$

Equations (F.11) and (F.10) together give

$$\tau_D \lambda^2 \frac{\int_0^1 \beta^2(s)\tau(s)^{-1} ds + \tau_\theta^{-1}}{1 - \lambda \int_0^1 \beta(s) ds} = \lambda \int_0^1 \beta(s) ds, \quad \text{(F.12)}$$

which implies that (F.9) reduces to (18a). Substituting (F.10) and (F.12) into (82) yields

$$\rho = \frac{\int_0^1 \beta(s)ds}{\int_0^1 \beta^2(s)\tau(s)^{-1}ds + \tau_\theta^{-1}}. \tag{F.13}$$

Using (18a) to express the integrals in (F.13) proves (F.6). From (F.13) it also follows that $\rho$ and $\int_0^1 \beta(s)ds$ must have the same sign. Finally, the second-order condition of trader $a$ is still satisfied if and only if (88) holds. Using the moments in (75), (78), and (F.8) to calculate the conditional variance shows, after some algebra, that (89) continues to hold, and thus as $da \to 0$ the the second-order condition of trader $a$ is satisfied if and only if $2\rho + \delta(a) > 0$. ■

**Theorem F.3** *Theorem 9 continues to hold with pure noise traders, with the only difference that Equation* (28a) *becomes*

$$\rho = \left[\int_0^1 \frac{\psi(s)}{[\delta(s)+2\rho]^3}ds + \frac{1}{\tau_\theta}\left(\rho\int_0^1 \beta(s)ds + \tau_D\right)\right]^{-1} \int_0^1 \frac{\psi(s)}{[\delta(s)+2\rho]^2}ds. \tag{F.14}$$

*and that the price is as in* (F.4).

**Proof.** Lemma 8 continues to hold with pure noise traders because $dz_a$ and $\theta$ are independent. Theorem F.3 follows by combining (F.6) of Theorem F.2 with (27) of Lemma 8.

■

**Proof of Corollary F.1 .** By Theorem F.3, with homogeneous risk aversion, $\delta(s) = \delta > 0$ for all $s$, the equilibrium is

$$\rho\left[\int_0^1 \psi(s)ds + \tau_\theta^{-1}\left(\rho\int_0^1 \beta(s)ds + \tau_D\right)(\delta+2\rho)^3\right] - (\delta+2\rho)\int_0^1 \psi(s)ds = 0, \tag{F.15a}$$

and

$$\int_0^1 \beta(s)ds\left[\rho\int_0^1 \beta(s)ds + \tau_D\right](\delta+2\rho)^2 - \int_0^1 \psi(s)ds = 0. \tag{F.15b}$$

Solving (F.15a) for $\int_0^1 \beta(s)ds$, and substituting the result into (F.15b) gives two equations, which after some algebra can be simplified to

$$\int_0^1 \beta(s)ds = \tau_\theta^{-1}\rho\frac{\delta+2\rho}{\delta+\rho}, \tag{F.16a}$$

and

$$\rho\left(\delta+2\rho\right)^3\tau_\theta^{-1}\left[\tau_D\left(\delta+\rho\right)+\tau_\theta^{-1}\rho^2\left(\delta+2\rho\right)\right]-\left(\delta+\rho\right)^2\int_0^1 \psi(s)ds = 0. \tag{F.16b}$$

By inspection it follows that Equation (F.16b) is a seventh order polynomial in $\rho$, in which the coefficients of orders seven through three are all positive. The signs of coefficients of orders two, one, and zero are as in the following table.

| term | $\rho^2$ | $\rho^1$ | $\rho^0$ |
|---|---|---|---|
| coefficient | $7\delta^3\tau_D\tau_\theta^{-1} - \int_0^1 \psi(s)ds$ | $\delta^4\tau_D\tau_\theta^{-1} - 2\delta\int_0^1 \psi(s)ds$ | $-\delta^2\int_0^1 \psi(s)ds$ |
| if $\delta^3\tau_D\tau_\theta^{-1} < \frac{1}{7}\int_0^1 \psi(s)ds$ | $-$ | $-$ | $-$ |
| if $\frac{1}{7}\int_0^1 \psi(s)ds < \delta^3\tau_D\tau_\theta^{-1} < 2\int_0^1 \psi(s)ds$ | $+$ | $-$ | $-$ |
| if $2\int_0^1 \psi(s)ds < \delta^3\tau_D\tau_\theta^{-1}$ | $+$ | $+$ | $-$ |

It follows that there is always a unique sign switch in the coefficients of $\rho$ in descending order, and thus, by Descartes' rule of signs, there exists a unique positive solution for $\rho$. By (F.16a) we also get that exists a unique positive solution for $\int_0^1 \beta(s)ds$, and thus there exists a unique equilibrium.

Next, by applying the implicit function theorem on (F.16b) we obtain

$$\frac{d\rho}{d\tau_\theta^{-1}} = -\frac{\tau_\theta \rho \left(\delta+\rho\right)\left(\delta+2\rho\right)\left[\tau_D\left(\delta+\rho\right)+2\tau_\theta^{-1}\rho^2\left(\delta+2\rho\right)\right]}{\tau_D\left(\delta+\rho\right)\left(6\rho^2+8\delta\rho+\delta^2\right)+2\tau_\theta^{-1}\rho^2\left(\delta+2\rho\right)\left(10\rho^2+15\delta\rho+3\delta^2\right)}, \tag{F.17}$$

which is negative because $\delta > 0$ and $\rho > 0$ in equilibrium. Because $\nu$ affects $\rho$ only through $\tau_\theta^{-1}$, it now follows by the chain rule that $\rho$ is decreasing in $\nu$. Finally, from (F.16a) we obtain

$$\frac{\int_0^1 \beta(s)ds}{d\tau_\theta^{-1}} = \rho\frac{\delta+2\rho}{\delta+\rho} + \tau_\theta^{-1}\frac{2\rho^2+4\delta\rho+\delta^2}{\left(\delta+\rho\right)^2} > 0, \tag{F.18}$$

and thus by the chain rule it follows that $\int_0^1 \beta(s)ds$ is increasing in $\nu$.

By (18b) and (F.16a),

$$\frac{d}{d\psi}\left(\frac{1}{\lambda}\right) = \frac{\tau_D}{\rho^2}\left[\frac{\rho^2\left(\delta^2+4\delta\rho+2\rho^2\right)}{\tau_D\tau_\theta\left(\delta+\rho\right)^2} - 1\right]\frac{d\rho}{d\psi} \tag{F.19}$$

By the Implicit Function Theorem and the equilibrium condition in (F.16b) it can be shown that $\frac{d\rho}{d\psi} > 0$, and thus whether liquidity increases in $\psi$ or not depends on the sign of the square-bracketed term on the right-hand side of (F.19). In particular, when $\delta = 0$ the sign of $\frac{d}{d\psi}\left(\frac{1}{\lambda}\right)$ is the same as that of

$$2\frac{\rho^2}{\tau_D\tau_\theta} - 1. \tag{F.20}$$

Solving (F.20) for $\rho$ and substituting the result into the equilibrium condition in (F.16b) provides a sign for (F.20). Because (F.16b) is increasing in $\rho$ at its only positive root, (F.20) is negative in equilibrium if and only if the root of (F.20) in $\rho$ is to the right of the equilibrium solution for $\rho$, i.e. if evaluating (F.16b) at $\rho^2 = \frac{\tau_D\tau_\theta}{2}$ is positive. Equivalently, (F.20) is negative if and only if

$$\psi < \sqrt{2\tau_\theta\tau_D^5}. \tag{F.21}$$

It can further be shown that $\frac{d\rho}{d\delta} < 0$, and that in equilibrium the square-bracketed term in (F.19) decreases in $\delta$ (by the Implicit Function Theorem on (F.16b) to obtain $\frac{d\rho}{d\delta}$, and by total differentiation.) Consequently, the equilibrium root for $\rho$ moves to the left as $\delta$ increases, and, because the square-bracketed term in (F.19) becomes more negative as $\delta$ increases if it is already negative for $\delta = 0$, (F.21) is a sufficient condition for (F.19) to be negative. Thus (F.19) can be negative for some values of $\psi$ strictly higher than what satisfies (F.21), and even more so for higher $\delta$. ■

# G The continuum model as a limit

Here I derive the large economy as a limit of a finite economy. I set up the finite economy with a finite number of traders, which I then send to infinity, obtaining the economy of Section 3.

There are $N$ informed competitive rational traders and one representative market maker. For each $n = 1, \ldots, N$, trader $n$ corresponds to point $a_{n-1}$, where $a_{n-1}$, $n = 1, \ldots, N$, are points that partition the interval $[0, 1)$ such that $0 = a_0 < a_1 < a_2 < \ldots < a_{N-1} < a_N = 1$, with $\Delta a_n = a_n - a_{n-1}$. As the partition becomes finer and finer by increasing $N$, it will eventually converge to a continuum of traders in the interval $[0, 1)$. To simplify the exposition I make the points equidistant, that is, $\Delta a_n = a_n - a_{n-1} = \Delta a = 1/N$ for each $n$.

I note that the double-subscript notation of $a_{n-1}$—rather than $n$—is not superfluous; it is rather a technical necessity so that the limit as $N \to \infty$ yields well-defined stochastic integrals in the sense of Itô (see, for example, Øksendahl (2003) p. 24, for details.) Nevertheless, where it is clear enough to do so, I use $n$ as an index instead of $a_{n-1}$.

Trader $n$ is endowed with a signal $\Delta z_{a_n}$ about the liquidating dividend, where

$$\Delta z_{a_n} = D\Delta a + \sqrt{\tau_{a_{n-1}}^{-1}}\Delta B_{a_n}. \tag{G.1}$$

Here, $B$ is a standard Brownian motion in the interval $[0, 1]$, independent of $D$, and $\Delta B_{a_n}$ is the *nth* Brownian increment,

$$\Delta B_{a_n} = B_{a_n} - B_{a_{n-1}}. \tag{G.2}$$

By independence of Brownian increments it follows that the signal noise of a particular trader is independent of the signal noise of all other traders.

The demand of trader $n$ is $X_{a_{n-1}}$. The market maker sets the price $P$ equal to his conditional expectation of the dividend given the aggregate order flow,

$$P = \mathbb{E}\left[D \middle| \sum_{i=1}^{N} X_{a_{i-1}}\right]. \tag{G.3}$$

I conjecture that demand strategies are linear in signals

$$X_{a_{n-1}} = \beta_{a_{n-1}} \Delta z_{a_n}, \tag{G.4}$$

and that the price is linear in aggregate order flow,

$$P = \lambda \sum_{i=1}^{N} X_{a_{i-1}}. \tag{G.5}$$

The profit for trader $n$ is $\pi_n = X_{a_{n-1}}(D - P)$, and his utility is

$$u\left(\pi_n; \Delta z_{a_n}\right) = \mathbb{E}\left[\pi_n \middle| \Delta z_{a_n}\right] - \frac{\delta_{a_{n-1}}}{2} \mathrm{Var}\left(\pi_n \middle| \Delta z_{a_n}\right), \tag{G.6}$$

where $\delta_{a_{n-1}}$ is his risk-preference coefficient.

Demand diversity is the discrete analogue of Equation (16), that is

$$\mathcal{V}_N = \sum_{i=1}^{N} X^2_{a_{i-1}}. \tag{G.7}$$

To relate trading volume in my economy to the volume in Admati and Pfleiderer (1988) and Foster and Viswanathan (1990), I define

$$\mathcal{E}_N = \max\left\{\left[\sum_{i=1}^{N} \left(X^+_{a_{i-1}}\right)^2\right]^{\frac{1}{2}}, \left[\sum_{i=1}^{N} \left(X^-_{a_{i-1}}\right)^2\right]^{\frac{1}{2}}\right\}, \tag{G.8}$$

which I call "Euclidean volume". Echoing the relation in Admati and Pfleiderer (1988), $\mathcal{E}_N$ is the maximum of the total orders on either side of the market, where instead of using the sums of the positive and negative parts of traders demands we use a metric based on the two-norm.[24]

It is straightforward to show that

$$X_{a_{n-1}} = \frac{\mathbb{E}\left[D - P_{-n} \middle| \Delta z_{a_n}\right]}{2\lambda + \delta_{a_{n-1}} \mathrm{Var}\left(D - P_{-n} \middle| \Delta z_{a_n}\right)}, \tag{G.9}$$

where $P_{-n} = \lambda \sum_{\substack{i=1 \\ i \neq n}}^{N} X_{a_{i-1}}$ is the price excluding the demand of trader $n$. This demand function, which is the discrete analogue of (73), follows the intuition that each strategic trader sets his demand by responding to the aggregation of strategies of other traders.

As the number of traders becomes large, the price without the impact of trader $n$, $P_{-n}$, converges to the price $P$.[25] If, in addition, $\lambda$ converges to zero, then the optimal demand of each trader approaches that of a price-taker. In such a case, negative risk aversion is troublesome, because the second-order condition with respect to demand is violated. It thus becomes important that there is a residual degree of market impact with a large number of traders, which following the intuition in Kyle (1989), we may describe as an information-based version of monopolistic competition.

Writing out the price function with the signal in (G.1) I get

$$P = D \sum_{i=1}^{N} \lambda \beta_{a_{n-1}} \Delta a + \sum_{i=1}^{N} \lambda \beta_{a_{n-1}} \sqrt{\tau_{a_{n-1}}^{-1}} \Delta B_{a_n}. \tag{G.10}$$

[24] We can think of the volume in Admati and Pfleiderer (1988) as using the taxicab norm to measure positive and negative orders (the positive and negative parts of a function are the same as the absolute values of the negative and positive parts.) In this light, we can think of the expression in (G.8) as using the Euclidean norm to measure positive and negative orders.

[25] More precisely, the difference between the $P$ and $P_{-n}$ becomes infinitesimally small.

As we can see in (G.10), the price contains two different kinds of summation, with different limiting behaviors. As the number of agents becomes large, the partition $a_{n-1}$, $n = 1, \ldots, N$ of the interval $[0, 1)$ converges to a continuum. Therefore, the sequence $\tau_{a_{n-1}}$, $n = 1, \ldots, N$ converges to a continuous function $\tau(a)$ on $[0, 1)$, and similarly, the sequence $\beta_{a_{n-1}}$, $n = 1, \ldots, N$ converges to a continuous function $\beta(a)$ on $[0, 1)$. These facts imply that the summation in the coefficient of the dividend $D$ in (G.10) converges to

$$\int_0^1 \lambda\beta(a)da, \tag{G.11}$$

in the sense of Riemann. The second summation in (G.10) is proportional to the aggregate price noise, and as we make the number of traders large it converges to

$$\int_0^1 \lambda\beta(a)\sqrt{\tau(a)^{-1}}dB_a. \tag{G.12}$$

Due to the presence of this aggregate noise, the impact that the aggregate order has on the price is larger than in models where the aggregated idiosycratic noises vanish in the limit. Recalling that market impact is zero in models where aggregated idiosyncratic noises vanish, we expect that in this model the market-impact parameter is positive. Of course, this property by itself is but the first step for a noisy equilibrium with large numbers of traders. Nevertheless, as the next result shows, there is a formal connection with this model and the continuum model, which we know is able to generate noisy prices.

**Theorem 17** *As the number of traders becomes large, the above economy converges to the economy of Section 3. In particular, the market-impact parameter and the trading intensity converge to the quantities that solve the system in Theorem 4, and the price converges to that in* (21)*. Moreover, the demand diversity converges to $\mathcal{V}$, and the Euclidean volume converges* *to a deterministic limit $\mathcal{E}$, which satisfies*

$$\frac{1}{2}\mathcal{V} \leq \mathcal{E}^2 \leq \mathcal{V}. \tag{G.13}$$

Theorem 17 says that we can think of the model in this section as a discretized version of the continuum economy. Moreover, the economy of Section 3 has very different implications for market impact than several microstructure models with competition. It is well known that the information structure employed by extant models of competition in the Kyle (1985) framework implies that in the large limit the market-impact parameter converges to zero, and the price converges to the dividend. In contrast, as we can see above, the market-impact parameter in my model converges to a positive constant.

I next elaborate on the connection with the monopolistic-competition limit in section 9 of Kyle (1989). It is nevertheless important to stress that Kyle (1989)'s monopolistic competition has solid theoretic justifications. As García and Sangiorgi (2011) show in detail, that model arises endogenously under information sales.

**Lemma 18** *Let $\breve{\varepsilon}_n$, $n = 1, \ldots, N$, be a collection of independent random variables, where $\breve{\varepsilon}_n$ has distribution $\mathcal{N}\left(0, \breve{\tau}_n^{-1}\right)$ and is independent of $D$. The economy with the signals in* (G.1) *is equivalent to an economy in which, for $n = 1, \ldots, N$, the signals are*

$$\breve{z}_n = D + \breve{\varepsilon}_n, \tag{G.14}$$

*with $\breve{\tau}_n = \tau_{a_{n-1}}\Delta a$, and the demand coefficients are $\breve{\beta}_n$ with $\breve{\beta}_n = \beta_{a_{n-1}}\Delta a$. For either signal representation, traders' demands, the aggregate signal, the price, and the market-impact parameter are identical.*

The two signals in (G.1) and (G.14) appear to be quite different, because the former uses the cross-sectional partition size $\Delta a$, whereas the latter is the standard way of representing a signal in the existing literature. Nevertheless, both signals represent exactly the same economy.[26] Before I explain why in more detail, I note that representation (G.1) has the advantage of being similar to an Itô process; we might as well call it an "Itô signal." As Lemma 18 shows, the signals in (G.1) and (G.14) are equivalent. This happens because they are different ways of writing the same thing:

$$\breve{\beta}_n \left(D + \varepsilon_n\right) = X_n = \beta_{a_{n-1}} \left(D + \varepsilon_n\right) \Delta a. \tag{G.15}$$

This fact allows us to cast (G.14), the usual way of representing signals, in the form of (G.1). We can therefore think of the economy herein as in the same class of economies as Holden and Subrahmanyam (1992), with the one-period version of Holden and Subrahmanyam (1992) being a special case of the Itô-signal economy for which every trader is risk-neutral and has infinite signal precision.

To summarize, no matter which representation we choose for the agent signal, the demand is always as in (G.15), where we can see that there is a $\Delta a$ incorporated into the demand coefficient. This, of course, is no accident. After all, $\Delta a = 1/N$, which says that the demand coefficient in (G.15) is inversely related to the number of agents in the economy. This is a straightforward effect of competition. As the number of agents increases, each agent has to trade less aggressively so as not to reveal too much to the market maker, which naturally decreases the magnitude of his demand. Another way to frame this effect comes from the standard intuition that competition erodes profits. Because profits are proportional to squared demands, increasing the number of traders decreases the magnitude of demand of any given trader.

[26] A summary intuition for this fact is that the two signal representations have the same signal-to-noise ratio, which is free of $\Delta a$.

In this light, what the signal representation of (G.1) achieves is to bake the scale of competition, $1/N$, directly into the agent signal. This now provides the mathematical representation of signals with $\Delta a$ in them with a solid economic justification. As I have proved above, in order to have aggregate noise in the large-economy limit, it suffices that the scale of the variance of individuals' noises is of order $\Delta a$. This is in fact the defining characteristic of Brownian motion. The economic takeaway is that in order to have aggregate price noise in a large market, the variance of the individuals' noises must scale directly with the scale of competition, similarly to a point made in García and Urošević (2013).

## G.1 Proofs

### G.1.1 Auxiliary results

**Lemma G.1** *The optimal demand is*

$$X_{a_{n-1}} = \frac{\mathbb{E}\left[D - P_{-n}\middle|\Delta z_{a_n}\right]}{2\lambda + \delta_{a_{n-1}}\, Var\left(D - P_{-n}\middle|\Delta z_{a_n}\right)}, \tag{G.16a}$$

*and the second-order condition of trader $n$ is satisfied if and only if*

$$\delta_{a_{n-1}} + \frac{2\lambda}{Var\left(D - P_{-n}\middle|\Delta z_{a_n}\right)} > 0. \tag{G.16b}$$

*Moreover, the utility of the optimal profit is*

$$u\left(\pi_n; \Delta z_{a_n}\right) = \frac{1}{2} X_{a_{n-1}} \mathbb{E}\left[D - P_{-n}\middle|\Delta z_{a_n}\right], \tag{G.16c}$$

*and the market-impact parameter is*

$$\lambda = \frac{Cov\left(D, \sum_{i=1}^{N} X_{a_{i-1}}\right)}{Var\left(\sum_{i=1}^{N} X_{a_{i-1}}\right)}. \tag{G.16d}$$

**Proof.** Due to assumptions (G.4) and (G.5) I can write the utility function as

$$u\left(\pi_n; \Delta z_{a_n}\right) = X_{a_{n-1}} \left\{\mathbb{E}\left[D - P_{-n} \middle| \Delta z_{a_n}\right] - \lambda X_{a_{n-1}}\right\} - \frac{\delta_{a_{n-1}}}{2} X_{a_{n-1}}^2 \text{Var}\left(D - P_{-n} \middle| \Delta z_{a_n}\right). \tag{G.17}$$

The first-order condition with respect to demand proves (G.16a). The second-order condition with respect to demand is

$$-\left[2\lambda + \delta_{a_{n-1}} \text{Var}\left(D - P_{-n} \middle| \Delta z_{a_n}\right)\right]. \tag{G.18}$$

Because $\text{Var}\left(D - P_{-n} \middle| \Delta z_{a_n}\right) > 0$, the second-order condition is negative if and only if (G.16b) holds. Combining (G.16a) with (G.17) proves (G.16c). Finally, from (G.3) we get

$$P = \mathbb{E}\left[D \middle| \sum_{i=1}^{N} X_{a_{i-1}}\right] = \frac{\text{Cov}\left(D, \sum_{i=1}^{N} X_{a_{i-1}}\right)}{\text{Var}\left(\sum_{i=1}^{N} X_{a_{i-1}}\right)} \sum_{i=1}^{N} X_{a_{i-1}}. \tag{G.19}$$

Matching the coefficient of aggregate demand in (G.19) with that in conjecture (G.5) we obtain (G.16d). ∎

**Proposition G.2** *For the economy of section G,* (G.4) *and* (G.5) *hold with*

$$\lambda = \frac{\frac{1}{\tau_D} \sum_{i=1}^{N} \beta_{a_{i-1}} \Delta a}{\frac{1}{\tau_D}\left(\sum_{i=1}^{N} \beta_{a_{i-1}} \Delta a\right)^2 + \sum_{i=1}^{N} \frac{\beta_{a_{i-1}}^2}{\tau_{a_{i-1}}} \Delta a}, \tag{G.20a}$$

*and*

$$\beta_{a_{n-1}} = \frac{\tau_{a_{n-1}}}{\delta_{a_{n-1}} + \frac{2\lambda\tau_D}{1-\lambda\sum_{i=1}^{N}\beta_{a_{i-1}}\Delta a} + A\left(\lambda, \tau_{a_{n-1}}, \beta_{a_0}, \ldots, \beta_{a_{n-1}}\right)\lambda\Delta a}, \tag{G.20b}$$

*where*

$$A\left(\lambda, \tau_{a_{n-1}}, \beta_{a_0}, \ldots, \beta_{a_{n-1}}\right) = \delta_{a_{n-1}}\left(2\beta_{a_{n-1}} + \frac{\tau_{a_{n-1}}}{\tau_D}\sum_{i=1}^{N}\beta_{a_{i-1}}\Delta a\right) + \frac{\tau_{a_{n-1}} - \delta_{a_{n-1}}\lambda\beta_{a_{n-1}}^2\frac{\tau_D}{\tau_{a_{n-1}}}}{1-\lambda\sum_{i=1}^{N}\beta_{a_{i-1}}\Delta a}. \tag{G.20c}$$

**Proof.** By Lemma G.1, conjectures (G.4) and (G.5), and due to that

$$\mathbb{E}\left[X_{a_{i-1}}\middle|\Delta z_{a_n}\right] = \mathbb{E}\left[D\middle|\Delta z_{a_n}\right]\beta_{a_{i-1}}\Delta a \tag{G.21}$$

because $\Delta z_{a_n}$ is independent of $\Delta B_{a_i}$ for $i \neq n$, it follows that

$$X_{a_{n-1}} = \frac{\left(1-\lambda\sum_{\substack{i=1\\i\neq n}}^{N}\beta_{a_{i-1}}\Delta a\right)\mathbb{E}\left[D\middle|\Delta z_{a_n}\right]}{2\lambda + \delta_{a_{n-1}}\left[\left(1-\lambda\sum_{\substack{i=1\\i\neq n}}^{N}\beta_{a_{i-1}}\Delta a\right)^2\mathrm{Var}\left(D\middle|\Delta z_{a_n}\right) + \lambda^2\left(\sum_{\substack{i=1\\i\neq n}}^{N}\frac{\beta_{a_{i-1}}^2}{\tau_{a_{i-1}}}\Delta a\right)\right]}. \tag{G.22}$$

The projection theorem implies

$$\mathbb{E}\left[D\middle|\Delta z_{a_n}\right] = \frac{\mathrm{Cov}\left(D, \Delta z_{a_n}\right)}{\mathrm{Var}\left(\Delta z_{a_n}\right)}\Delta z_{a_n} = \frac{\tau_{a_{n-1}}}{\tau_{a_{n-1}}\Delta a + \tau_D}\Delta z_{a_n}, \tag{G.23}$$

and

$$\mathrm{Var}\left(D\middle|\Delta z_{a_n}\right) = \mathrm{Var}\left(D\right) - \frac{\mathrm{Cov}\left(D, \Delta z_{a_n}\right)^2}{\mathrm{Var}\left(\Delta z_{a_n}\right)} = \frac{1}{\tau_{a_{n-1}}\Delta a + \tau_D}. \tag{G.24}$$

Substituting (G.23) and (G.24) into (G.22) and matching with our conjecture about linear demands gives

$$\beta_{a_{n-1}} = \frac{\tau_{a_{n-1}}}{\tau_{a_{n-1}}\Delta a + \tau_D} \frac{1 - \lambda \sum_{\substack{i=1 \\ i\neq n}}^{N} \beta_{a_{i-1}} \Delta a}{2\lambda + \delta_{a_{n-1}} \left[ \left(1 - \lambda \sum_{\substack{i=1 \\ i\neq n}}^{N} \beta_{a_{i-1}} \Delta a\right)^2 \frac{1}{\tau_{a_{n-1}}\Delta a + \tau_D} + \lambda^2 \left( \sum_{\substack{i=1 \\ i\neq n}}^{N} \frac{\beta_{a_{i-1}}^2}{\tau_{a_{i-1}}} \Delta a \right) \right]}$$

$$= \frac{\tau_{a_{n-1}} \left(1 - \lambda \sum_{i=1}^{N} \beta_{a_{i-1}} \Delta a + \lambda \beta_{a_{n-1}} \Delta a\right)}{2\lambda\left(\tau_{a_{n-1}}\Delta a + \tau_D\right) + \delta_{a_{n-1}} \left[ \left(1 - \lambda \sum_{\substack{i=1 \\ i\neq n}}^{N} \beta_{a_{i-1}} \Delta a\right)^2 + \lambda^2 \left( \sum_{\substack{i=1 \\ i\neq n}}^{N} \frac{\beta_{a_{i-1}}^2}{\tau_{a_{i-1}}} \Delta a \right) \left(\tau_{a_{n-1}}\Delta a + \tau_D\right) \right]}, \tag{G.25}$$

and rearranging yields

$$\beta_{a_{n-1}} = \frac{\tau_{a_{n-1}} \left(1 - \lambda \sum_{i=1}^{N} \beta_{a_{i-1}} \Delta a\right)}{\lambda\left(\tau_{a_{n-1}}\Delta a + 2\tau_D\right) + \delta_{a_{n-1}} \left[ \left(1 - \lambda \sum_{\substack{i=1 \\ i\neq n}}^{N} \beta_{a_{i-1}} \Delta a\right)^2 + \lambda^2 \left( \sum_{\substack{i=1 \\ i\neq n}}^{N} \frac{\beta_{a_{i-1}}^2}{\tau_{a_{i-1}}} \Delta a \right) \left(\tau_{a_{n-1}}\Delta a + \tau_D\right) \right]}. \tag{G.26}$$

Substituting (G.4) into (G.16d) gives

$$\lambda = \frac{\text{Cov}\left(D, \sum_{i=1}^{N} X_{a_{i-1}}\right)}{\text{Var}\left(\sum_{i=1}^{N} X_{a_{i-1}}\right)} = \frac{\frac{1}{\tau_D} \sum_{i=1}^{N} \beta_{a_{i-1}} \Delta a}{\frac{1}{\tau_D} \left(\sum_{i=1}^{N} \beta_{a_{i-1}} \Delta a\right)^2 + \sum_{i=1}^{N} \frac{\beta_{a_{i-1}}^2}{\tau_{a_{i-1}}} \Delta a}, \tag{G.27}$$

which establishes (G.20a). From this equation we also get

$$\lambda^2 \left( \sum_{i=1}^{N} \frac{\beta_{a_{i-1}}^2}{\tau_{a_{i-1}}} \Delta a \right) = \lambda \frac{1}{\tau_D} \sum_{i=1}^{N} \beta_{a_{i-1}} \Delta a - \lambda^2 \frac{1}{\tau_D} \left( \sum_{i=1}^{N} \beta_{a_{i-1}} \Delta a \right)^2 . \tag{G.28}$$

Using Equation (G.28) we can rewrite the term multiplying $\delta_{a_{n-1}}$ in the denominator of (G.26) as

$$\begin{aligned}
&\left(1 - \lambda \sum_{i=1}^{N} \beta_{a_{i-1}} \Delta a + \lambda \beta_{a_{n-1}} \Delta a \right)^2 + \lambda^2 \left( -\frac{\beta_{a_{n-1}}^2}{\tau_{a_{n-1}}} \Delta a + \sum_{i=1}^{N} \frac{\beta_{a_{i-1}}^2}{\tau_{a_{i-1}}} \Delta a \right) \left( \tau_{a_{n-1}} \Delta a + \tau_D \right) \\
&= \left(1 - \lambda \sum_{i=1}^{N} \beta_{a_{i-1}} \Delta a \right)^2 + 2 \left(1 - \lambda \sum_{i=1}^{N} \beta_{a_{i-1}} \Delta a \right) \left( \lambda \beta_{a_{n-1}} \Delta a \right) + \left( \lambda \beta_{a_{n-1}} \Delta a \right)^2 \\
&\quad - \lambda^2 \frac{\beta_{a_{n-1}}^2}{\tau_{a_{n-1}}} \Delta a \left( \tau_{a_{n-1}} \Delta a + \tau_D \right) + \lambda^2 \left( \sum_{i=1}^{N} \frac{\beta_{a_{i-1}}^2}{\tau_{a_{i-1}}} \Delta a \right) \left( \tau_{a_{n-1}} \Delta a + \tau_D \right) \\
&= 1 - \lambda \sum_{i=1}^{N} \beta_{a_{i-1}} \Delta a \\
&\quad + \left[ \lambda \left(1 - \lambda \sum_{i=1}^{N} \beta_{a_{i-1}} \Delta a \right) \left( 2\beta_{a_{n-1}} + \frac{\tau_{a_{n-1}}}{\tau_D} \sum_{i=1}^{N} \beta_{a_{i-1}} \Delta a \right) - \lambda^2 \beta_{a_{n-1}}^2 \frac{\tau_D}{\tau_{a_{n-1}}} \right] \Delta a,
\end{aligned} \tag{G.29}$$

and therefore (G.26) becomes

$$\begin{aligned}
\beta_{a_{n-1}} = \tau_{a_{n-1}} &\left(1 - \lambda \sum_{i=1}^{N} \beta_{a_{i-1}} \Delta a \right) \cdot \\
&\left\{ 2\lambda \tau_D + \delta_{a_{n-1}} \left(1 - \lambda \sum_{i=1}^{N} \beta_{a_{i-1}} \Delta a \right) + \lambda \left[ \tau_{a_{n-1}} - \delta_{a_{n-1}} \lambda \beta_{a_{n-1}}^2 \frac{\tau_D}{\tau_{a_{n-1}}} \right.\right. \\
&\left.\left. + \delta_{a_{n-1}} \left(1 - \lambda \sum_{i=1}^{N} \beta_{a_{i-1}} \Delta a \right) \left( 2\beta_{a_{n-1}} + \frac{\tau_{a_{n-1}}}{\tau_D} \sum_{i=1}^{N} \beta_{a_{i-1}} \Delta a \right) \right] \Delta a \right\}^{-1} .
\end{aligned} \tag{G.30}$$

This proves (G.20b) and (G.20c). ■

**Lemma G.3**

$$\frac{1}{2}\sum_{i=1}^{N} X_{a_{i-1}}^2 \le \mathcal{E}_N^2 \le \sum_{i=1}^{N} X_{a_{i-1}}^2. \tag{G.31}$$

**Proof.** For any positive numbers $Y$ and $Z$ we have

$$\left(\max\left\{\sqrt{Y}, \sqrt{Z}\right\}\right)^2 = \max\left\{Y, Z\right\}. \tag{G.32}$$

Moreover, for any random variable $X$ we have

$$\left(X^+\right)^2 = \left[\frac{1}{2}\left(|X| + X\right)\right]^2 = \frac{1}{4}\left(|X|^2 + X^2 + 2|X|X\right) = \frac{1}{2}X^2 + \frac{1}{2}|X|X, \tag{G.33a}$$

and

$$\left(X^-\right)^2 = \left[\frac{1}{2}\left(|X| - X\right)\right]^2 = \frac{1}{4}\left(|X|^2 + X^2 - 2|X|X\right) = \frac{1}{2}X^2 - \frac{1}{2}|X|X. \tag{G.33b}$$

Applying (G.32) to definition (G.8) with

$$Y = \sum_{i=1}^{N}\left(X_{a_{i-1}}^+\right)^2 \tag{G.34}$$

and

$$Z = \sum_{i=1}^{N}\left(X_{a_{i-1}}^-\right)^2, \tag{G.35}$$

we get

$$\begin{aligned}\mathcal{E}_N^2 &= \max\left\{\sum_{i=1}^{N}\left(X_{a_{i-1}}^+\right)^2, \sum_{i=1}^{N}\left(X_{a_{i-1}}^-\right)^2\right\}\\ &= \frac{1}{2}\max\left\{\sum_{i=1}^{N} X_{a_{i-1}}^2 + \sum_{i=1}^{N}|X_{a_{i-1}}|X_{a_{i-1}}, \sum_{i=1}^{N} X_{a_{i-1}}^2 - \sum_{i=1}^{N}|X_{a_{i-1}}|X_{a_{i-1}}\right\}\end{aligned}$$

$$= \frac{1}{2}\sum_{i=1}^{N} X_{a_{i-1}}^2 + \frac{1}{2}\max\left\{\sum_{i=1}^{N} |X_{a_{i-1}}|X_{a_{i-1}}, -\sum_{i=1}^{N} |X_{a_{i-1}}|X_{a_{i-1}}\right\}$$

$$= \frac{1}{2}\sum_{i=1}^{N} X_{a_{i-1}}^2 + \frac{1}{2}\left|\sum_{i=1}^{N} |X_{a_{i-1}}|X_{a_{i-1}}\right|, \quad \text{(G.36)}$$

where the second equality follows by the properties in (G.33). Relation (G.31) now follows. In particular, the upper bound follows by the triangle inequality, and the lower bound follows because the absolute value in (G.36) is bounded below by zero. ■

### G.1.2 Main proofs for the continuum as a limit

**Proof of Theorem 17.** As $N \to \infty$, the partition $a_{n-1}$, $n = 1, \dots, N$ of the interval $[0, 1)$ converges to a continuum. Therefore, the sequence $\tau_{a_{n-1}}$, $n = 1, \dots, N$ converges to a continuous function $\tau(a)$ on $[0, 1)$, and similarly, the sequence $\delta_{a_{n-1}}$, $n = 1, \dots, N$ converges to a continuous function $\delta(a)$ on $[0, 1)$. Similarly, $\beta_{a_{n-1}}$, $n = 1, \dots, N$ also converges to a continuous function on $[0, 1)$. Let $\beta_\star$ denote this limit. In addition, let $\lambda_\star$ denote the limit of $\lambda$.

By inspection of (G.20a) and (G.20b) of Proposition G.2, as $N \to \infty$ we obtain

$$\lambda_\star = \frac{\int_0^1 \beta_\star(s)ds}{\left(\int_0^1 \beta_\star(s)ds\right)^2 + \tau_D\left(\int_0^1 \frac{\beta_\star^2(s)}{\tau(s)}ds\right)} \quad \text{(G.37)}$$

and

$$\beta_\star(a) = \frac{\tau(a)}{\delta(a) + 2\frac{\lambda_\star \tau_D}{1-\lambda_\star \int_0^1 \beta_\star(s)ds}}. \quad \text{(G.38)}$$

Letting

$$\rho_\star = \frac{\lambda_\star \tau_D}{1 - \lambda_\star \int_0^1 \beta_\star(s)ds} \quad \text{(G.39)}$$

shows that if $\rho_\star$ as defined in (G.39) is the solution to (18c), then $\beta_\star$ is the function $\beta$ in

(18a), and that $\lambda_\star$ is $\lambda$ in (18b). To wit, substituting (G.37) and (G.38) into (G.39) gives

$$\rho_\star = \left[\int_0^1 \frac{\tau(s)}{[2\rho_\star + \delta(s)]^2} ds\right]^{-1} \int_0^1 \frac{\tau(s)}{2\rho_\star + \delta(s)} ds, \tag{G.40}$$

and thus $\rho_\star$ is the solution to (18c).

Writing out the price function in (G.5) with the signal in (G.1) gives

$$P = D\lambda \sum_{i=1}^{N} \beta_{a_{n-1}} \Delta a + \lambda \sum_{i=1}^{N} \beta_{a_{n-1}} \sqrt{\tau_{a_{n-1}}^{-1}} \Delta B_{a_n}. \tag{G.41}$$

We have that

$$\lim_{N\to\infty} \sum_{i=1}^{N} \beta_{a_{n-1}} \Delta a = \int_0^1 \beta(a) da \tag{G.42}$$

in the sense of Riemann. By standard results in stochastic calculus (see, e.g., Øksendahl, 2003, ch. 3) we obtain that

$$\lim_{N\to\infty} \sum_{i=1}^{N} \beta_{a_{n-1}} \sqrt{\tau_{a_{n-1}}^{-1}} \Delta B_{a_n} = \int_0^1 \beta(a) \sqrt{\tau(a)^{-1}} dB_a \tag{G.43}$$

in the sense of Itô, and thus the price in (G.41) converges to the price in (21).

Given that the limit price is as in the continuum, the aggregate price noise $\mathcal{A}$ and the signal-to-noise ratio $\mathcal{Q}$ are identical to those for the continuum model.

For finite $N$, the demand diversity is

$$\begin{aligned}
\mathcal{V}_N = \sum_{n=1}^{N} X_{a_{n-1}}^2 &= \sum_{n=1}^{N} \beta_{a_{n-1}}^2 (\Delta z_{a_n})^2 \\
&= \sum_{n=1}^{N} \beta_{a_{n-1}}^2 \left(D^2 \Delta a^2 + 2D\Delta a \sqrt{\tau_{a_{n-1}}^{-1}} \Delta B_{a_n} + \tau_{a_{n-1}}^{-1} \Delta B_{a_n}^2\right)
\end{aligned}$$

$$= \Delta a D^2 \sum_{n=1}^{N} \beta_{a_{n-1}}^2 \Delta a + 2\Delta a D \sum_{n=1}^{N} \beta_{a_{n-1}}^2 \sqrt{\tau_{a_{n-1}}^{-1}} \Delta B_{a_n} + \sum_{n=1}^{N} \beta_{a_{n-1}}^2 \tau_{a_{n-1}}^{-1} \Delta B_{a_n}^2. \quad (G.44)$$

The sum

$$\sum_{n=1}^{N} \beta_{a_{n-1}}^2 \Delta a \quad (G.45)$$

converges to the Riemann integral

$$\int_0^1 \beta^2(a) da. \quad (G.46)$$

The sum

$$\sum_{n=1}^{N} \beta_{a_{n-1}}^2 \sqrt{\tau_{a_{n-1}}^{-1}} \Delta B_{a_n} \quad (G.47)$$

converges to the stochastic integral

$$\int_0^1 \frac{\beta^2(a)}{\sqrt{\tau(a)}} dB_a. \quad (G.48)$$

The sum

$$\sum_{n=1}^{N} \beta_{a_{n-1}}^2 \tau_{a_{n-1}}^{-1} \Delta B_{a_n}^2 \quad (G.49)$$

is a quadratic variation sum, so it converges to the Riemann integral

$$\int_0^1 \frac{\beta^2(a)}{\tau(a)} da. \quad (G.50)$$

Taking the limit in (G.44) therefore yields

$$\lim_{N\to\infty} \mathcal{V}_N = \left(\lim_{N\to\infty} \Delta a\right) D^2 \int_0^1 \beta^2(a) da + 2\left(\lim_{N\to\infty} \Delta a\right) D \int_0^1 \frac{\beta^2(a)}{\sqrt{\tau(a)}} dB_a + \int_0^1 \frac{\beta^2(a)}{\tau(a)} da$$
$$= \int_0^1 \frac{\beta^2(a)}{\tau(a)} da = \mathcal{V}. \quad (G.51)$$

By Lemma G.1 and Equation (G.21) for $i \neq n$, the utility of trader $n$ is

$$u\left(\pi_n; \Delta z_{a_n}\right) = \frac{1}{2}\left(1 - \lambda \sum_{\substack{i=1 \\ i \neq n}}^{N} \beta_{a_{i-1}} \Delta a\right) X_{a_{n-1}} \mathbb{E}\left[D \middle| \Delta z_{a_n}\right]. \tag{G.52}$$

This together with (G.4) implies that the ex-ante expectation of the utility is

$$\begin{aligned}
\mathbb{E}\left[u\left(\pi_n; \Delta z_{a_n}\right)\right] &= \frac{1}{2}\left(1 - \lambda \sum_{\substack{i=1 \\ i \neq n}}^{N} \beta_{a_{i-1}} \Delta a\right) \beta_{a_{n-1}} \mathbb{E}\left[\Delta z_{a_n} \mathbb{E}\left[D \middle| \Delta z_{a_n}\right]\right] \\
&= \frac{1}{2}\left(1 - \lambda \sum_{\substack{i=1 \\ i \neq n}}^{N} \beta_{a_{i-1}} \Delta a\right) \beta_{a_{n-1}} \frac{\tau_{a_{n-1}}}{\tau_{a_{n-1}} \Delta a + \tau_D} \mathbb{E}\left[\Delta z_{a_n}^2\right] \\
&= \frac{\beta_{a_{n-1}}}{2\tau_D}\left(1 - \lambda \sum_{\substack{i=1 \\ i \neq n}}^{N} \beta_{a_{i-1}} \Delta a\right) \Delta a \\
&= \left(1 - \lambda \sum_{i=1}^{N} \beta_{a_{i-1}} \Delta a\right) \frac{\beta_{a_{n-1}}}{2\tau_D} \Delta a + \lambda \frac{\beta_{a_{n-1}}^2}{2\tau_D} \left(\Delta a\right)^2 \to O(da)
\end{aligned} \tag{G.53}$$

This shows that in the limit the utility of each trader is of the order $da$, and that we need to adopt a differential notation for it, as in (72).

By Lemma G.3 and Jensen's inequality we obtain

$$\mathcal{E}_N^4 \leq \left(\sum_{i=1}^{N} X_{a_{i-1}}^2\right)^2 \leq \sum_{i=1}^{N} X_{a_{i-1}}^4. \tag{G.54}$$

By inspection of the demand function $X_{a_{i-1}}$, it follows that the sum $\sum_{i=1}^{N} X_{a_{i-1}}^4$ is of order $\Delta a$ (the lowest order in $\Delta a$ of each term in the sum is $\Delta B_{a_i}^4$, which is of order $(\Delta a)^2$.) By taking expectations in (G.54) we get

$$\mathbb{E}\left[\mathcal{E}_N^4\right] \leq \mathbb{E}\left[\sum_{i=1}^{N} X_{a_{i-1}}^4\right], \tag{G.55}$$

and, because the right-hand converges to zero as $N \to \infty$, it follows that the fourth moment of trading volume converges to zero. This proves that trading volume has a deterministic limit. Finally, the bounds in (G.13) follow from Lemma G.3 and from that, as established above,

$$\lim_{N\to\infty} \sum_{n=1}^{N} X_{a_{n-1}}^2 = \mathcal{V}. \tag{G.56}$$

■

**Proof of Lemma 18.** By inspection of Proposition G.2, the system (G.20) is the same as the system

$$\lambda = \frac{\frac{1}{\tau_D}\sum_{i=1}^{N}\beta_{a_{i-1}}\Delta a}{\frac{1}{\tau_D}\left(\sum_{i=1}^{N}\beta_{a_{i-1}}\Delta a\right)^2 + \sum_{i=1}^{N}\frac{\left(\beta_{a_{i-1}}\Delta a\right)^2}{\tau_{a_{i-1}}\Delta a}}, \tag{G.57a}$$

and

$$\beta_{a_{n-1}}\Delta a = \frac{\tau_{a_{n-1}}\Delta a}{\delta_{a_{n-1}} + \frac{2\lambda\tau_D}{1-\lambda\sum_{i=1}^{N}\beta_{a_{i-1}}\Delta a} + \breve{A}\left(\lambda, \tau_{a_{n-1}}\Delta a, \beta_{a_0}\Delta a, \ldots, \beta_{a_{n-1}}\Delta a\right)\lambda}, \tag{G.57b}$$

where

$$\breve{A}\left(\lambda, \tau_{a_{n-1}}\Delta a, \beta_{a_0}\Delta a, \ldots, \beta_{a_{n-1}}\Delta a\right) = \delta_{a_{n-1}}\left(2\beta_{a_{n-1}}\Delta a + \frac{\tau_{a_{n-1}}\Delta a}{\tau_D}\sum_{i=1}^{N}\beta_{a_{i-1}}\Delta a\right) + \frac{\tau_{a_{n-1}}\Delta a - \delta_{a_{n-1}}\lambda\left(\beta_{a_{i-1}}\Delta a\right)^2\frac{\tau_D}{\tau_{a_{n-1}}\Delta a}}{1-\lambda\sum_{i=1}^{N}\beta_{a_{i-1}}\Delta a}. \tag{G.57c}$$

Let $\breve{\tau}_n = \tau_{a_{n-1}}\Delta a$ and $\breve{\beta}_n = \beta_{a_{n-1}}\Delta a$ for each $n$. This change of variables does not affect system (G.57). The market impact, in particular, remains the same. To prove that everything else in the economy remains the same it suffices to show that $\breve{\beta}_n$ is the demand coefficient and $\breve{\tau}_n$ is the precision in the representation with signal (G.14). Writing out the demand function in the two representations we get

$$\begin{aligned} \beta_{a_{n-1}} \left( D\Delta a + \frac{1}{\sqrt{\tau_{a_{n-1}}}} \Delta B_{a_n} \right) = X_{a_{n-1}} = \breve{\beta}_n \left( D + \breve{\varepsilon}_n \right) \quad \Leftrightarrow \\ \beta_{a_{n-1}} \Delta a \left( D + \frac{\Delta B_{a_n}}{\Delta a \sqrt{\tau_{a_{n-1}}}} \right) = X_{a_{n-1}} = \breve{\beta}_n \left( D + \breve{\varepsilon}_n \right). \end{aligned} \tag{G.58}$$

By matching coefficients of the dividend and noise we get that $\breve{\beta}_n$ is the demand coefficient, and that

$$\mathrm{Var} \left( \frac{\Delta B_{a_n}}{\Delta a \sqrt{\tau_{a_{n-1}}}} \right) = \left( \tau_{a_{n-1}} \Delta a \right)^{-1} = \mathrm{Var} \left( \breve{\varepsilon}_n \right), \tag{G.59}$$

which shows that $\breve{\tau}_n = \tau_{a_{n-1}} \Delta a$. ■